



UNIMORE
UNIVERSITÀ DEGLI STUDI DI
MODENA E REGGIO EMILIA



Dipartimento di Ingegneria
“Enzo Ferrari”

Progettazione Assistita di Organi di Macchine

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Agenda

Castigliano's Theorem analysis of structures by Maxima:

- CASE D: Rollbar
- Centroid and Shear center
- Symmetry BCs
- Screw-symmetry BCs
- CASE E: Ladder frame
- References

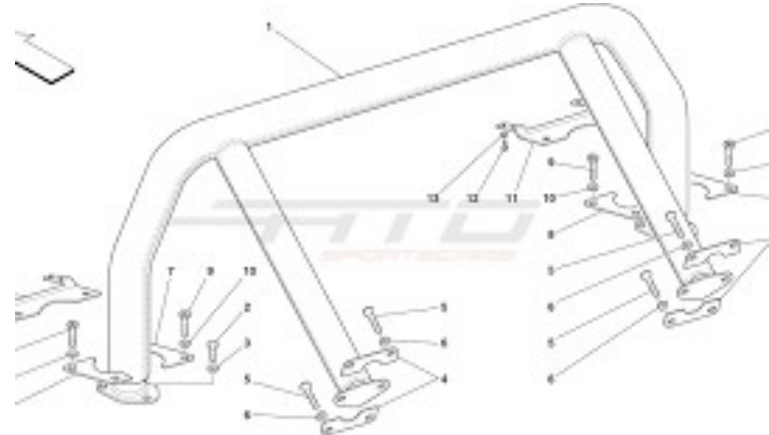
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Castigliano's Theorem

CASE D: Roll bar - Statically redundant structure (+3 dof)

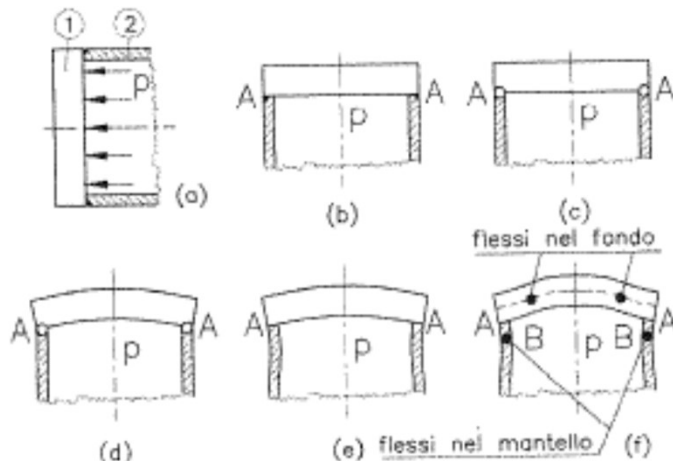
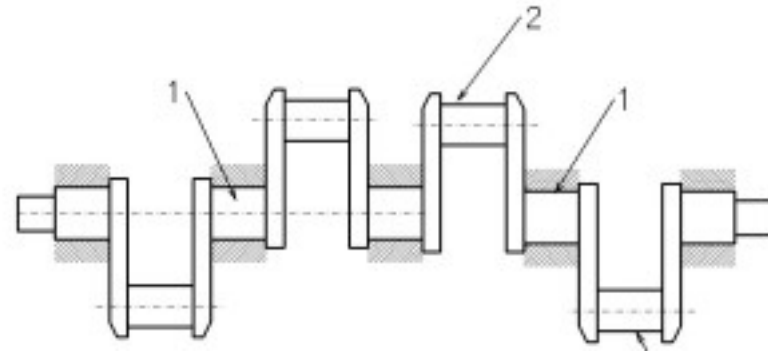


Castigliano's Theorem

Rollbar and further mechanical applications



Crankshaft throw

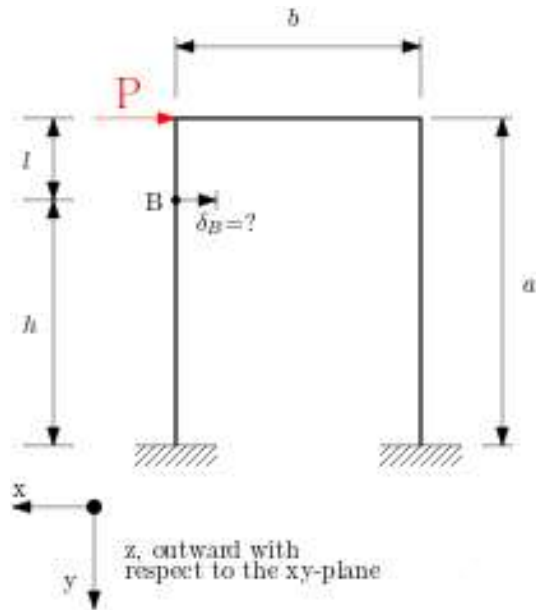


A pressurized vessel where the cylinder tube and the cap are welded (axisymmetric geometry).

Approximate to a plane frame omitting the pressure acting at the cylinder tube.

Castigliano's Theorem

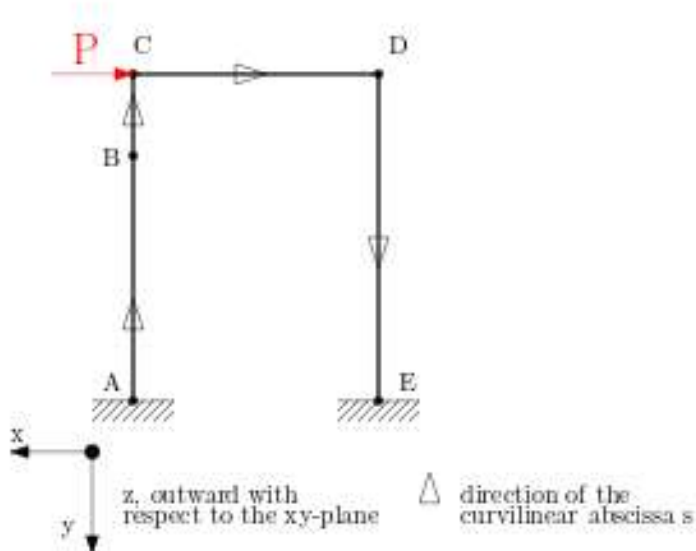
CASE D: Roll bar - Statically redundant structure (+3 dof)



Considering a simplified roll bar:

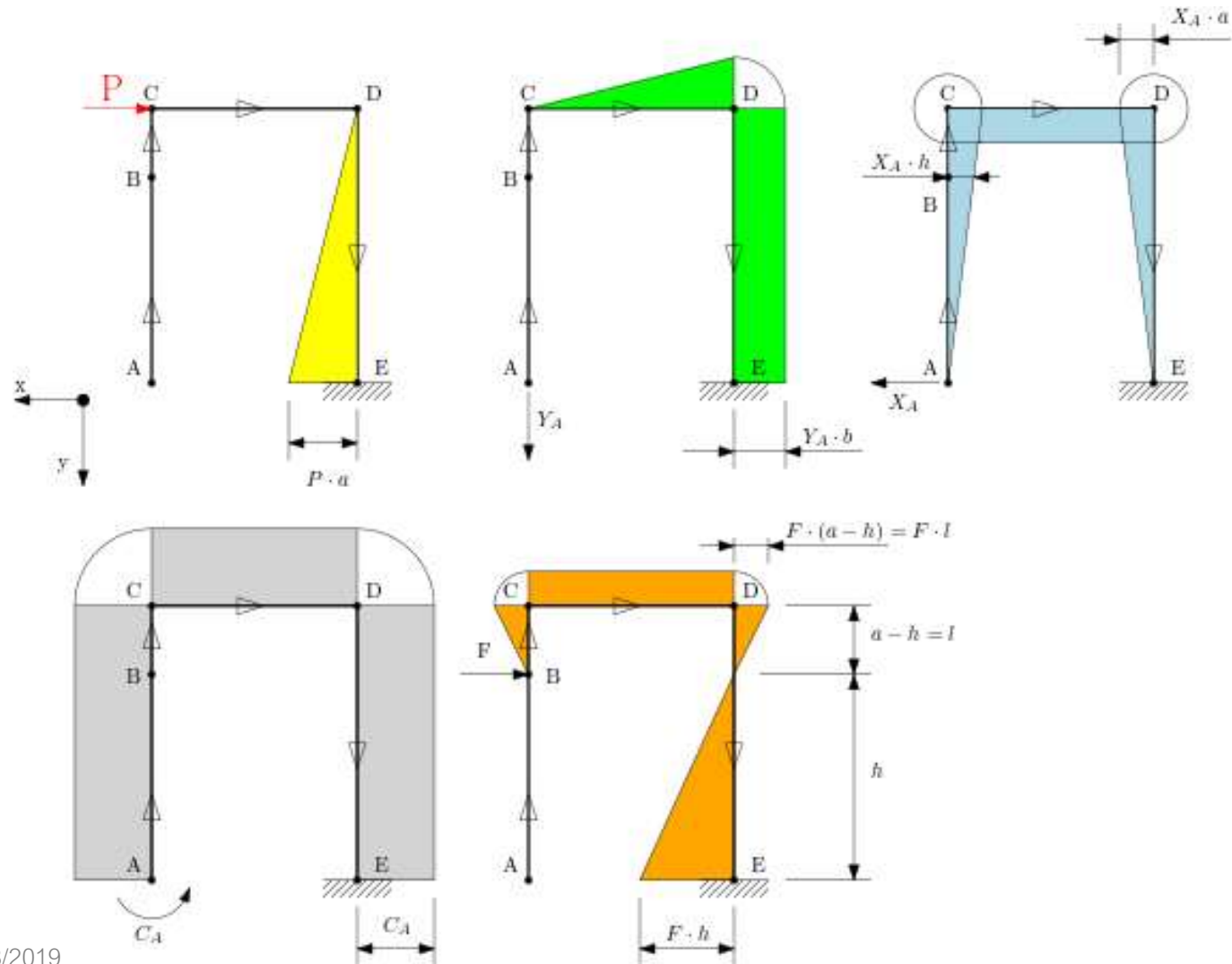
- fixed to the extremities;
- loaded by a lateral concentrated force (P) acting at the point B of the structure.

Evaluate the deflection (δ_B) acting at the point B of the structure, located at the maximum point at which the driver and the passenger can reach during a rollover crash event.



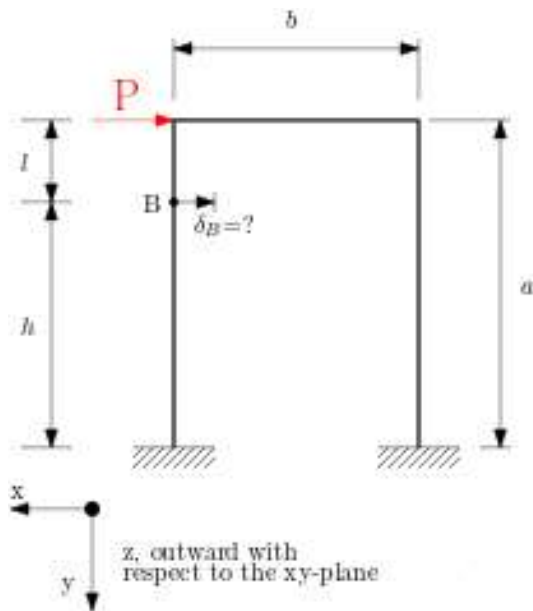
Castigliano's Theorem

CASE D: Roll bar - Statically redundant structure (+3 dof)



Castigliano's Theorem

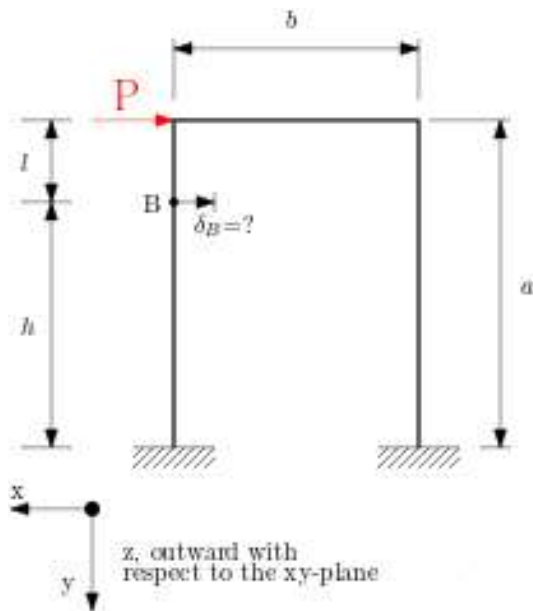
CASE D: Roll bar - Statically redundant structure (+3 dof)



1. Application of a fictitious force F at point B;
2. Evaluation of the equilibrium of the structure, to retrieve the reaction forces and moment acting at E;
3. Definition of a linear shape functions in the $[0,1]$ interval;
4. Definition of the bending moment acting on the portions of the structure, called as Mf_AB , Mf_BC , Mf_CD , Mf_DE ;
5. Definition of the elastic internal energy related to the various beam segments U_AB , U_BC , U_CD , U_DE ;
6. Evaluation of the total elastic internal energy of the structure defined as the sum of the various beam segments.

Castigliano's Theorem

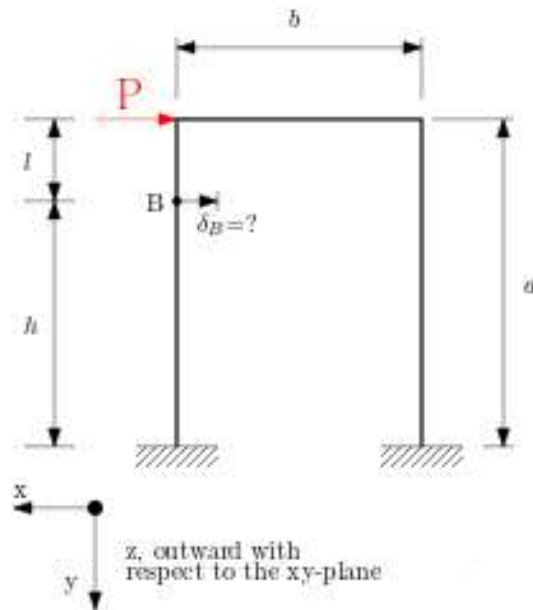
CASE D: Roll bar - Statically redundant structure (+3 dof)



7. Application of the Castigliano's theorem for obtaining the displacements and the rotation at A named as u_A , v_A , r_A .
8. Definition of kinematic congruence with respect to the clamp constraint in A is to be enforced, by the mean of a system of (linear) equations.
9. Evaluation of the redundant reaction force and moment acting at point A (X_A , Y_A , C_A) starting from the system of equation imposed by the kinematic congruence equations (see point 8).

Castigliano's Theorem

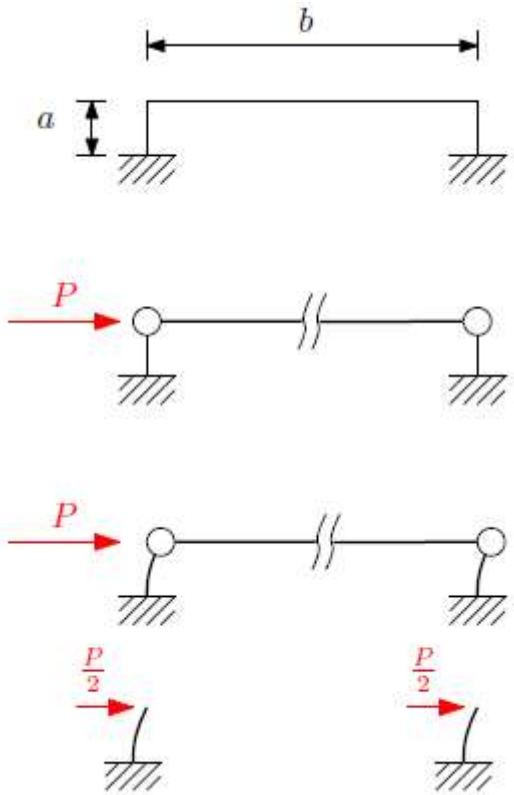
CASE D: Roll bar - Statically redundant structure (+3 dof)



10. Evaluation of the overall internal energy of the structure U , substituting the definition of X_A , Y_A , C_A . The U relation is function of the external load P and of the fictitious force F acting at B .
11. The displacement at the B point is evaluated through the Castigliano's theorem;
12. The fictitious nature of F may now be enforced to be null.

Castigliano's Theorem

Considerations Case 1: $b \gg a$



The horizontal beam shows high bending deformation, however it is rigid at the normal force, therefore it might be approximated by a rod.

The vertical elastic beams work on parallel and the external loading P is equally subdivided.

$$dC_{ref} = \frac{a^3}{3EJ} P;$$

$$K_{ref} = \frac{3EJ}{a^3}$$

Castigliano's Theorem

Considerations Case 2: $b \ll a$

The horizontal beam to its limited length, is rigid on the flexural deformation, therefore might be considered as a rigid body.

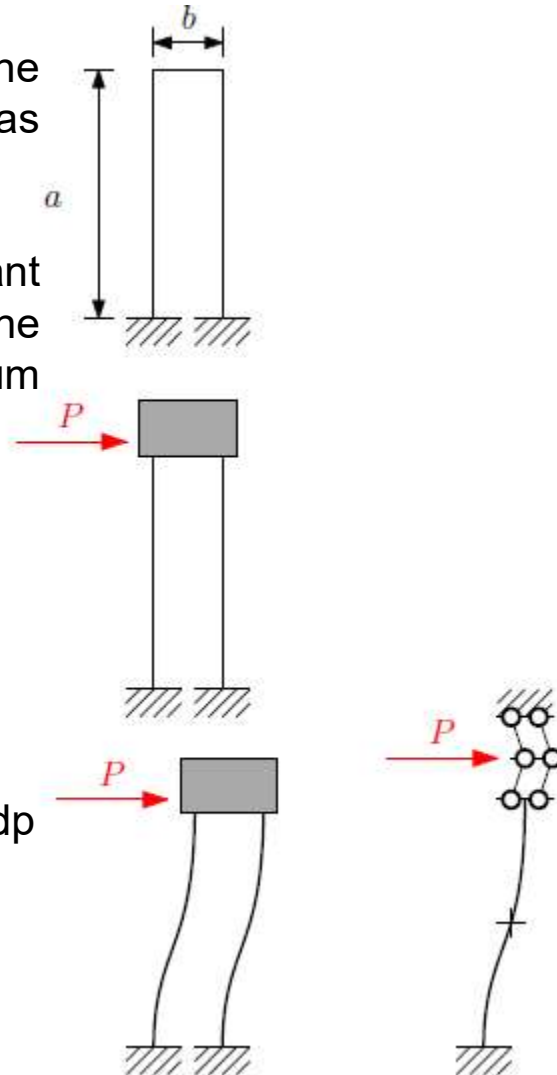
The vertical beams deforms similarly to a redundant beam structure (+1dof), fixed at one end and on the other end constraints with a double-double pendulum (ddp). The lateral external load at the latter extremity.

$$K_{cantilever} = \frac{3EJ}{a^3}$$

$$K_{fixed-ddp} = 4 * K_{cantilever}$$

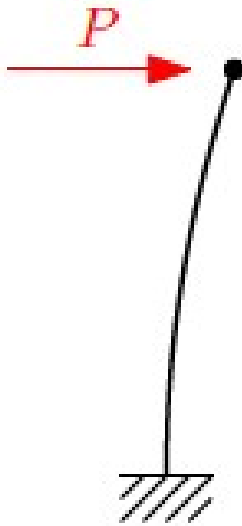
Therefore, the stiffness of the rollbar structure at this configuration might be rationalized as two beam with ddp and fixed BCs.

$$\begin{aligned} K_{rollbar} &= 2 * K_{fixed-ddp} = 2 * (4 * K_{cantilever}) \\ &= 8 * K_{cantilever} \end{aligned}$$



Castigliano's Theorem

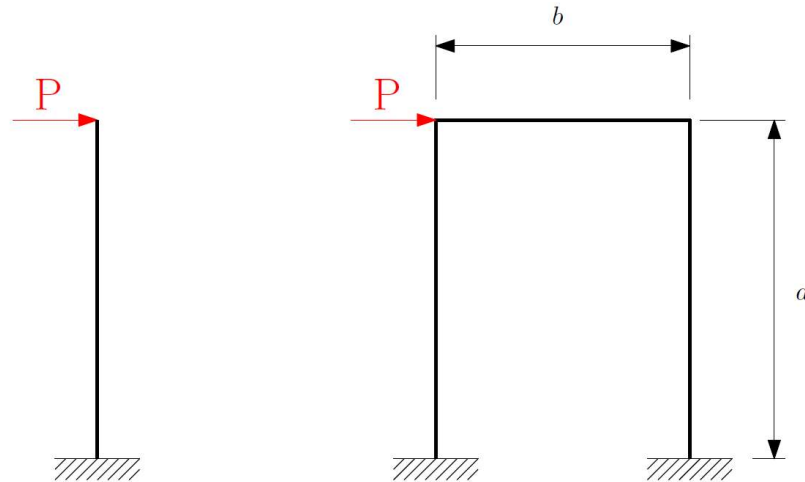
Considerations Case 3: $b = 0$



In this case, the two vertical beams coincide therefore this peculiar structure might be represented by a beam with stiffness doubles than the single cantilever beam.

Castigliano's Theorem

Considerations Case 3: $b = 0$



Cantilever beam
Reference model

Rollbar

$$dC_{ref} = \frac{a^3}{3EJ} P;$$

$$K_{ref} = \frac{3EJ}{a^3}$$

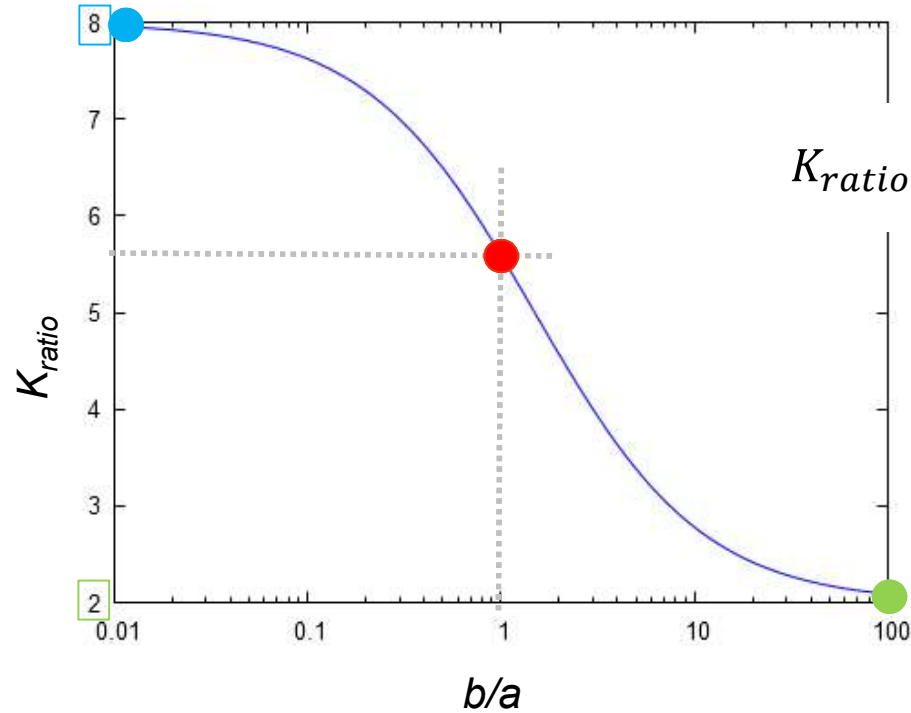
$$K = \frac{P}{dC}$$

$$K_{ratio} = \frac{K_{rollbar}}{K_{ref}} = \frac{dC_{ref}}{dC_{rollbar}} = \frac{dC_{ref}}{dC}$$

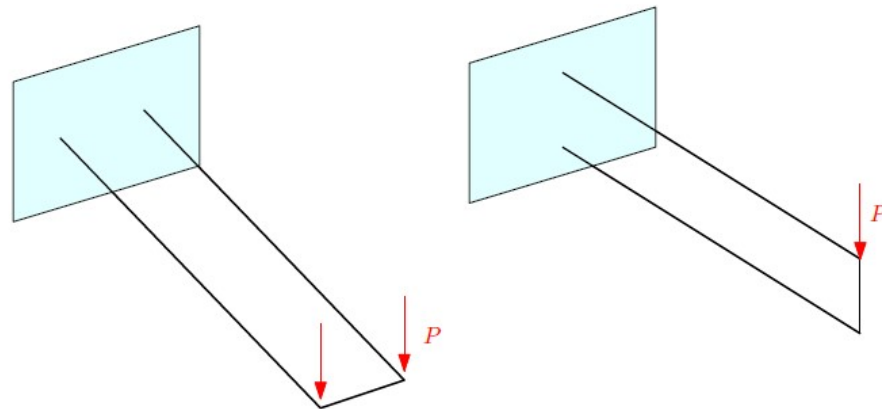
Where dC has been evaluated by the rollbar Maxima program.

Castigliano's Theorem

Considerations



$$K_{ratio} = \frac{K_{rollbar}}{K_{ref}} = \frac{dC_{ref}}{dC_{rollbar}} = \frac{dC_{ref}}{dC}$$



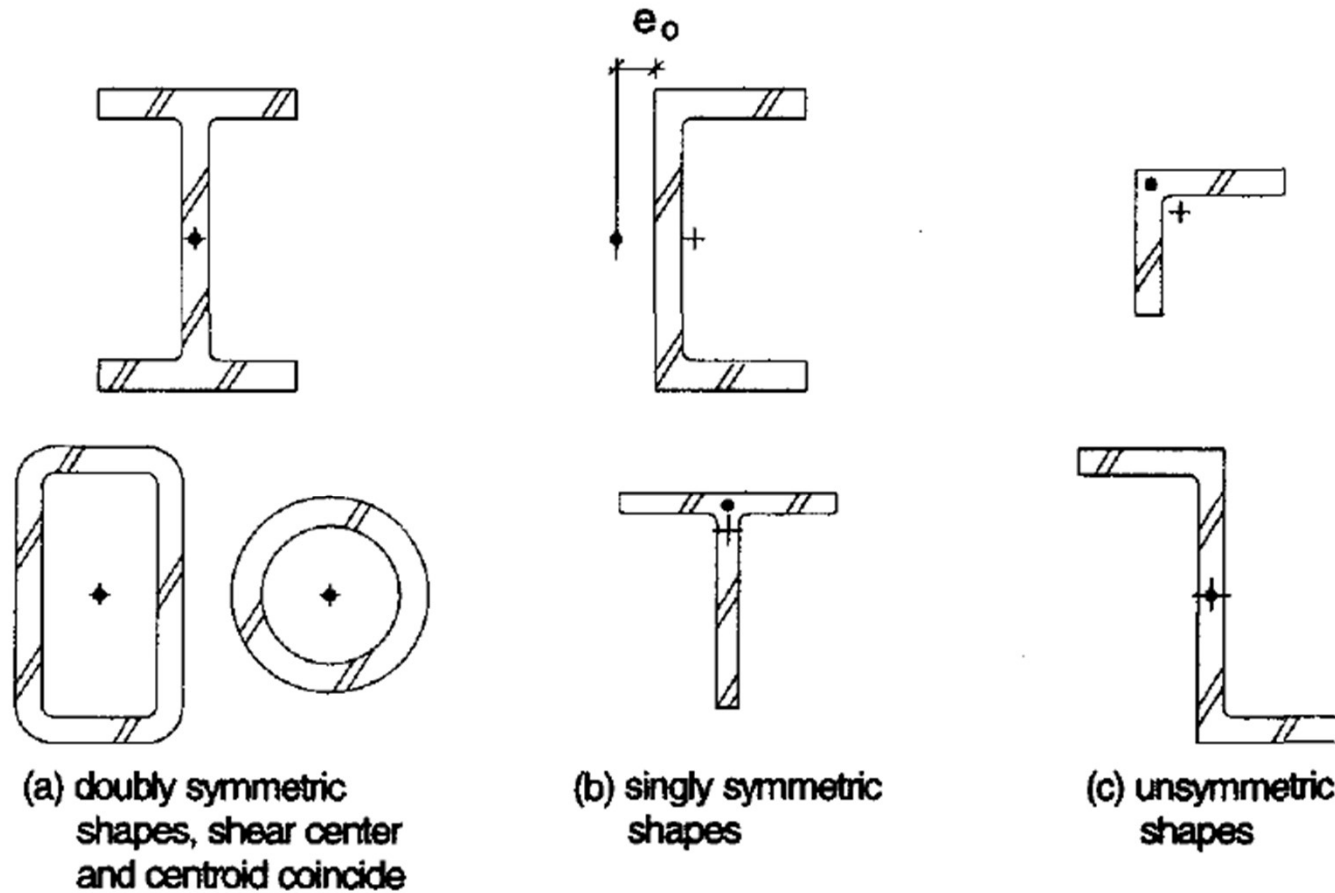
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Centroid and Shear center

Generic and peculiar cross-sections



+ centroid • shear center

Centroid and Shear center

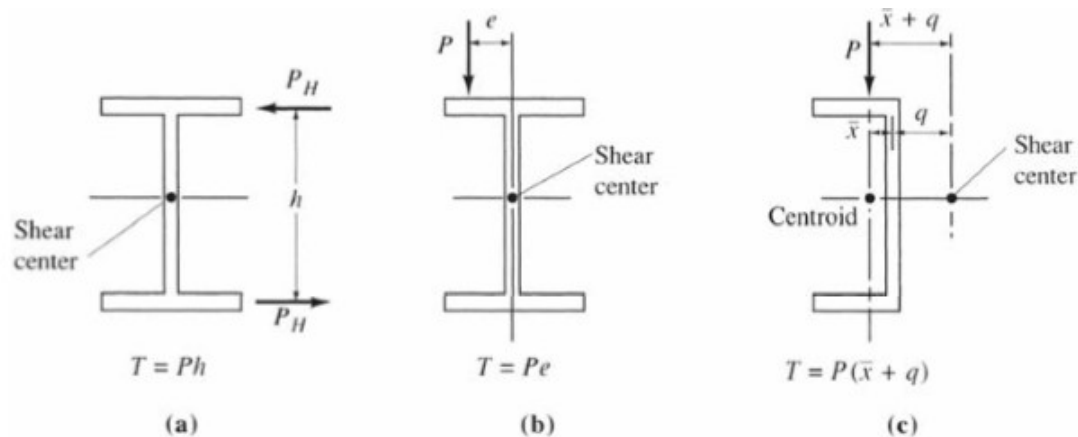
Generic and peculiar cross-sections

Cross sections with:

- 1) At least two planes of symmetry, the centroid and the shear center coincide;
- 2) Only one plane on symmetry, the centroid and the shear center lies on the axis of symmetry
→ shear and
 - 1) The generic section, the evaluation of the shear center is not a trivial structural problem
 - 2) Engineering books provide the shear center positioning for the more common cross-sections.

Centroid and Shear center

Generic and peculiar cross-sections



Se la forza tagliante passa per il centro di taglio (CT) essa produce solamente tensioni taglianti e non torsionali, ossia non produce una rotazione della trave rispetto al proprio asse.

Per sezioni:

- A due o infiniti assi di simmetria il baricentro ed il centro di taglio coincidono;
- ad un solo asse di simmetria ho una deformata torsionale sovrapposta ad una flessionale, se la forza non passa per il CT.

Se la forza P viene applicata lungo la retta d'azione della forza passa per il centro di taglio, la trave si inflette senza ruotare.

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Symmetry BCs

Why do you adopt symmetry?

In most cases, utilizing as many planes of symmetry as are allowed by the problem will result in shorter run times, more accurate boundary conditions, and more accurate solutions deriving from the previous two benefits.

Any 3D model can have a maximum of three orthogonal planes of symmetry in which the geometry, properties, and boundary conditions are equivalent across these planes.

An object has reflectional symmetry (line or mirror symmetry) if there is a line going through it which divides it into two pieces which are **mirror images** of each other.

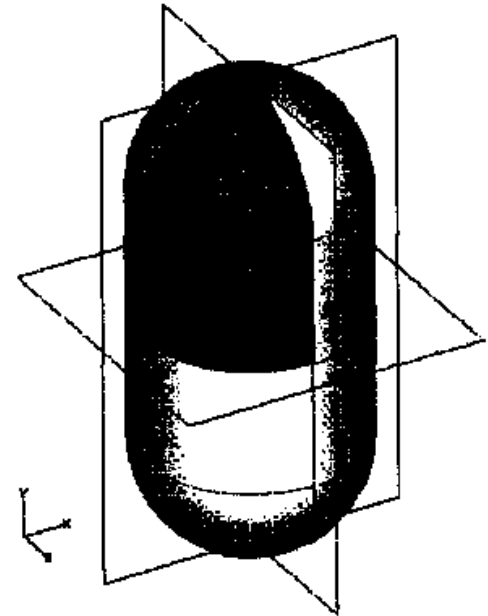


Fig. 4.15. Three symmetry planes for a domed pressure vessel with the section required for an internal pressure load.

Symmetry BCs

Why do you adopt symmetry?

Considering the three planes of symmetry, the vessel could be modelled with only one eighth of the structure.

Symmetry conditions require that the the geometry, and the boundary conditions are equal across one, two, or three planes.

→ Loading and constraints symmetry definition.

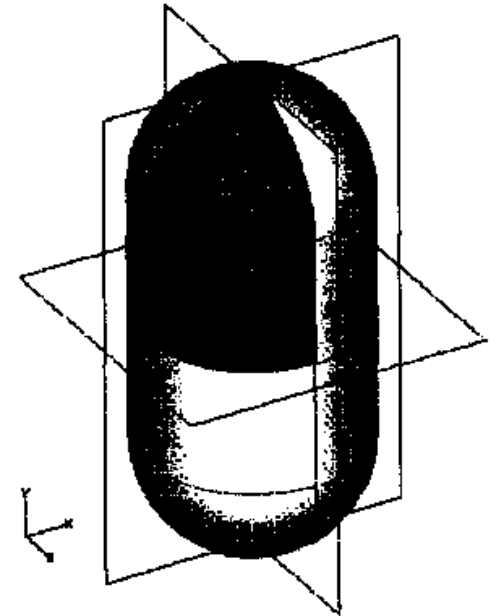


Fig. 4.15. Three symmetry planes for a domed pressure vessel with the section required for an internal pressure load.

Symmetry BCs

Loading

The loading applied to a symmetric model should be divided by the number of the symmetry planes used:

N. of planes	Magnitude of the symmetry loading condition
1	$\frac{1}{2} F$
2	$\frac{1}{4} F$
3	$\frac{1}{8} F$
$p = F/A$???

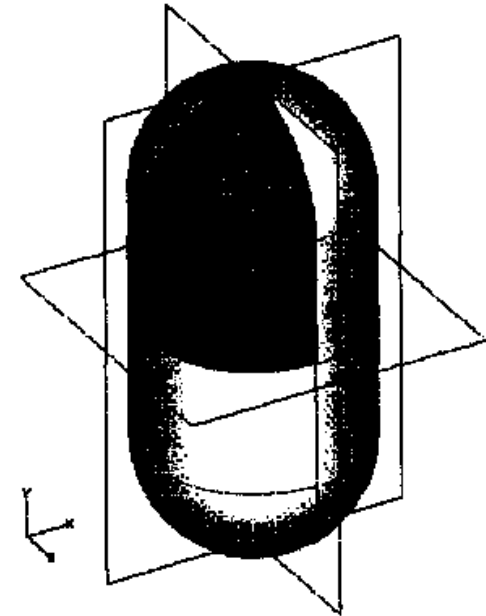


Fig. 4.15. Three symmetry planes for a domed pressure vessel with the section required for an internal pressure load.

A pressure load (p) will automatically halve or quarter themselves due to the available surface area considered in the symmetric model.

NOTE: a check of the total load and the magnitude of the loading to be applied for a symmetrical model could be thought by the mirror images concept.

Symmetry BCs

Constraints

The constraints on:

- 1) a **solid model** must prevent traslation throught the plane of symmetry on the entire cut face;
- 2) On beam and shell elements must also prevent rotation in the components parallel to the cut planes.

These constraints ensure tangency and continuity at the cut plane, just as the other half of the model would if it existed.

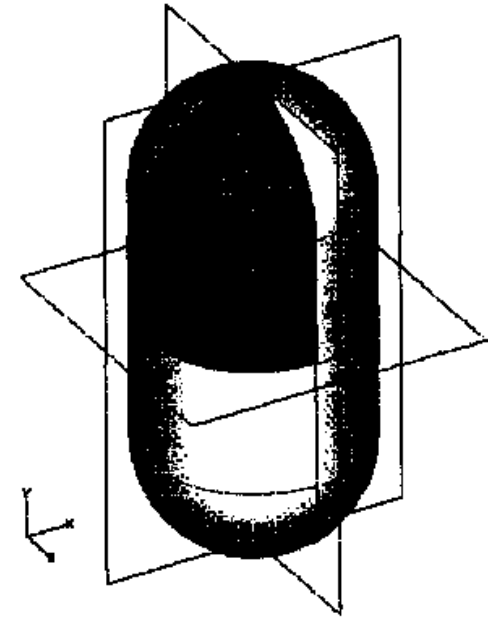


Fig. 4.15. Three symmetry planes for a domed pressure vessel with the section required for an internal pressure load.

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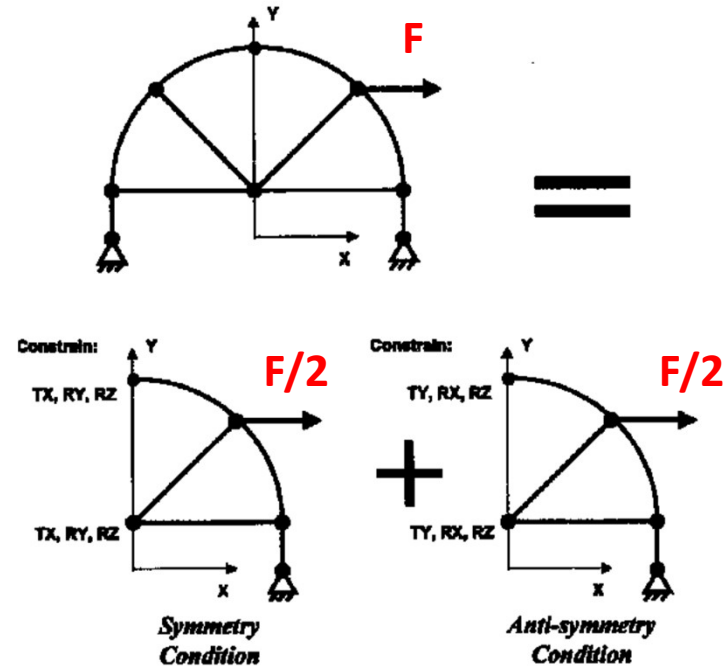
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Screw-symmetry BCs

What are these BCs?

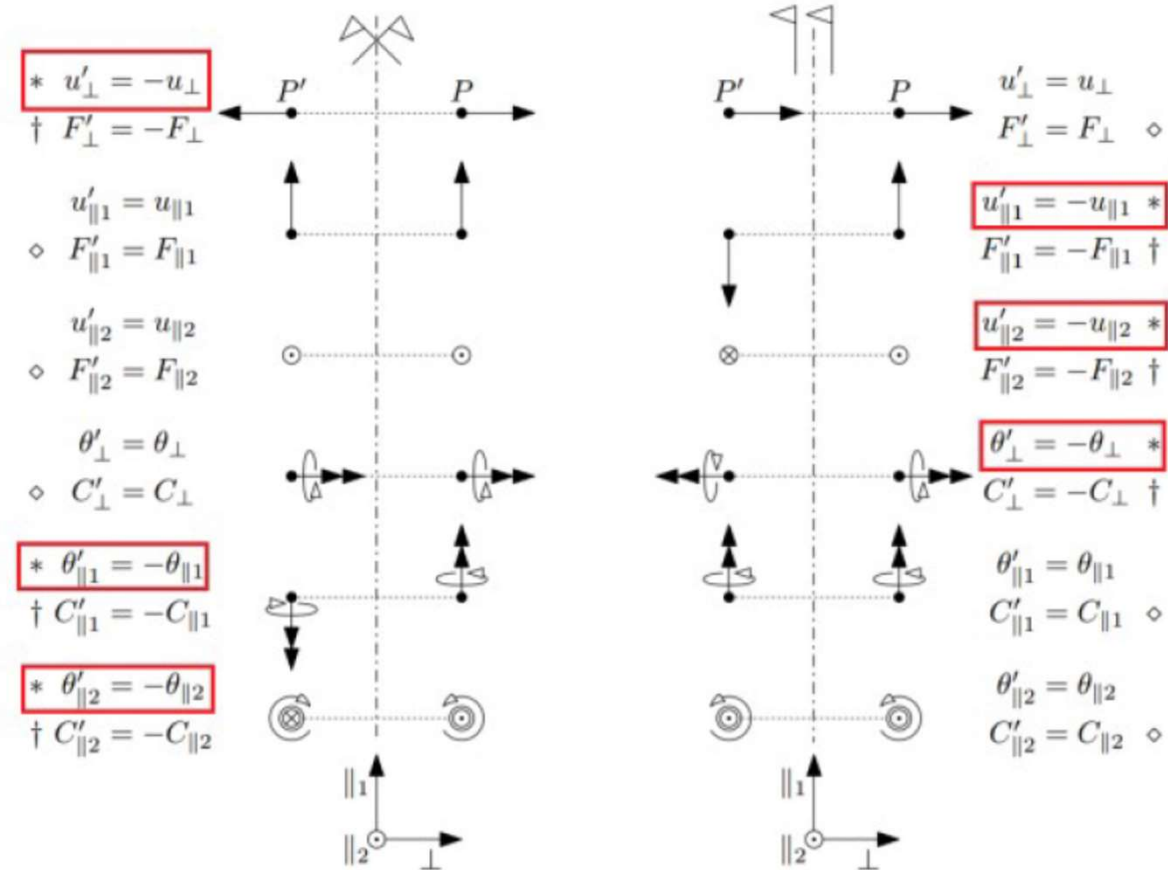
These constraints are not so intuitive as the previously described symmetry conditions.

This technique can be used when the geometry conforms to planar symmetry however the loading does not.



Symmetry and screw-symmetry BCs

A comparison



*: dof to be imposed as null.

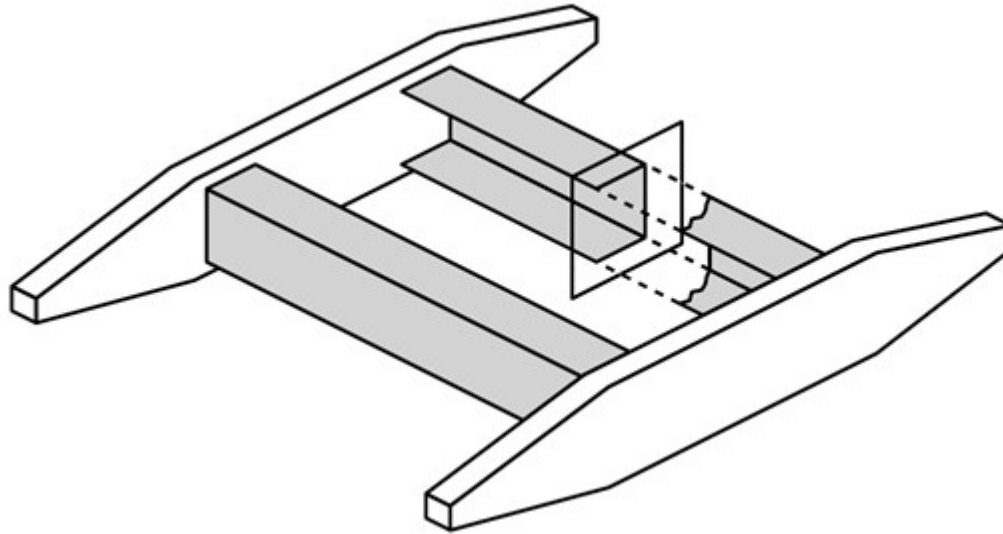
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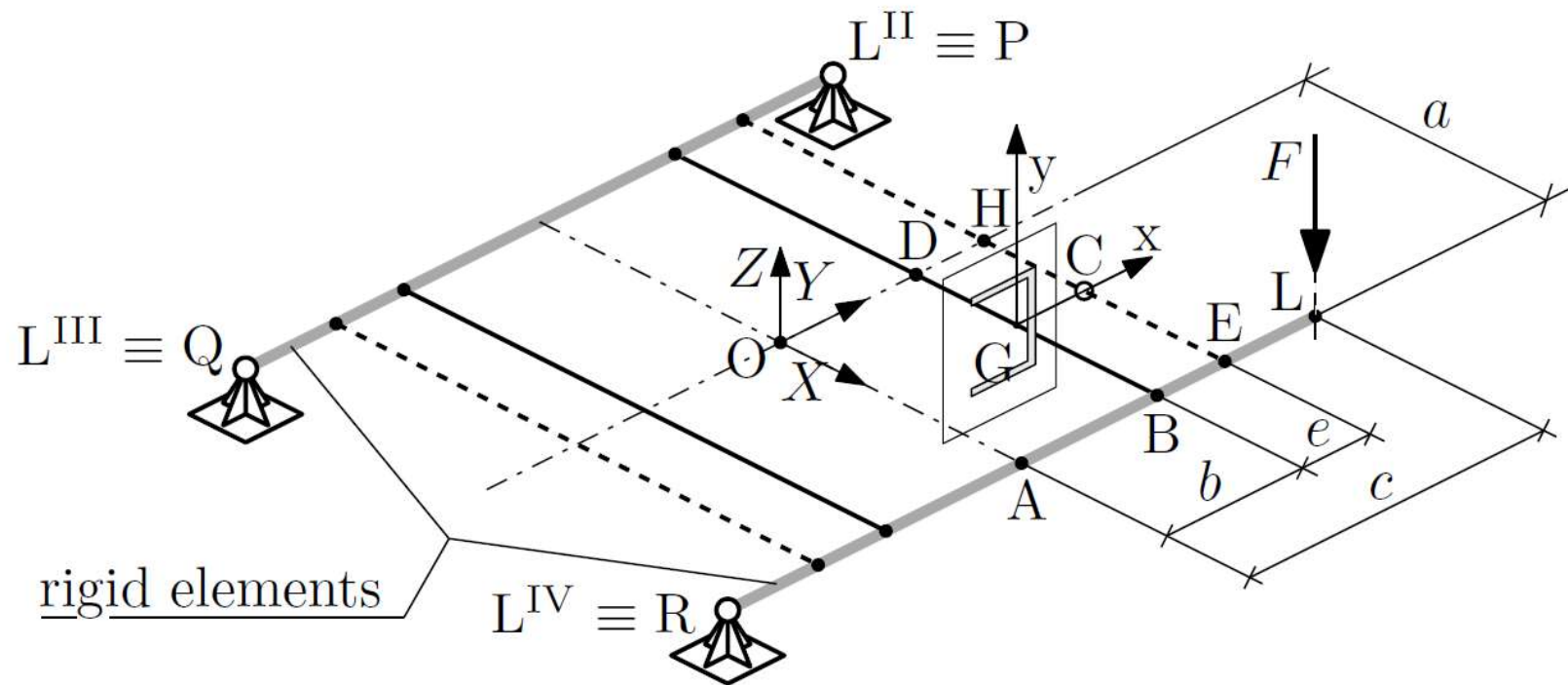
CASE E: Ladder Frame – Problem Introduction



Evaluate the torsional stiffness (t_s) of the chassis supported at three wheel centers and loaded at the fourth adopting Castigliano's theorem.

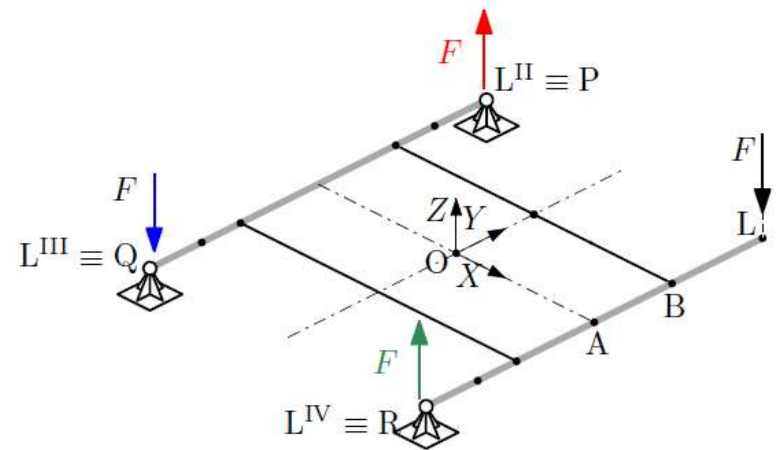
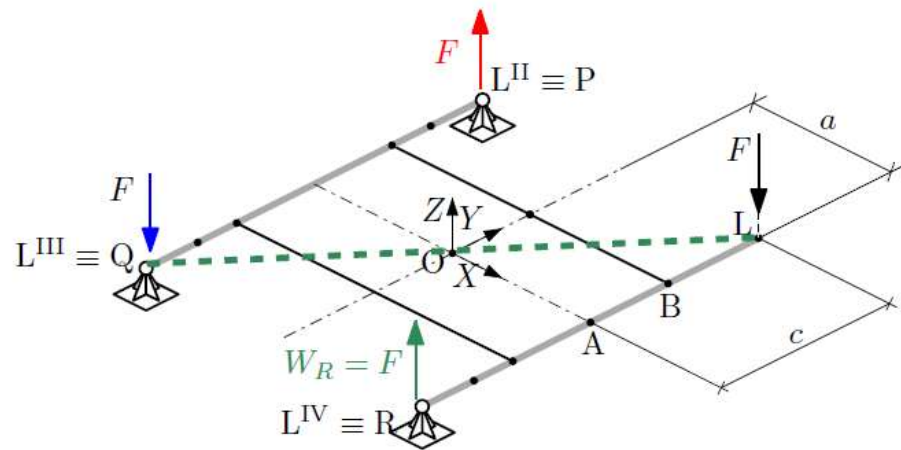
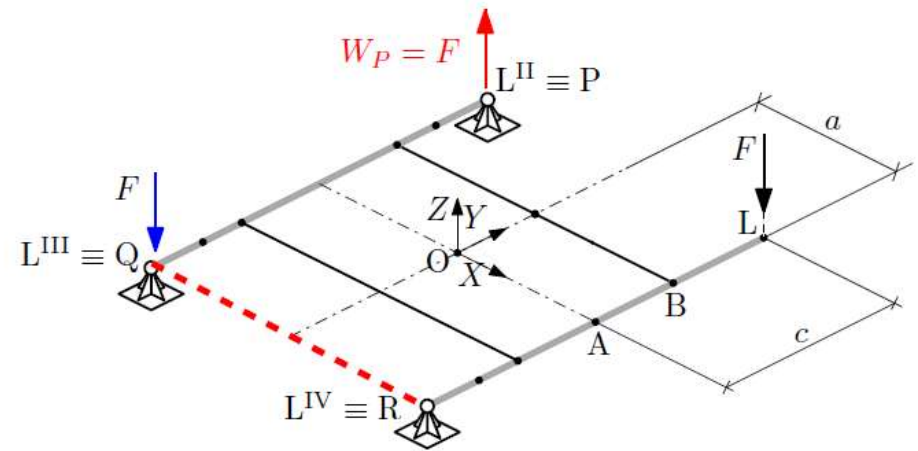
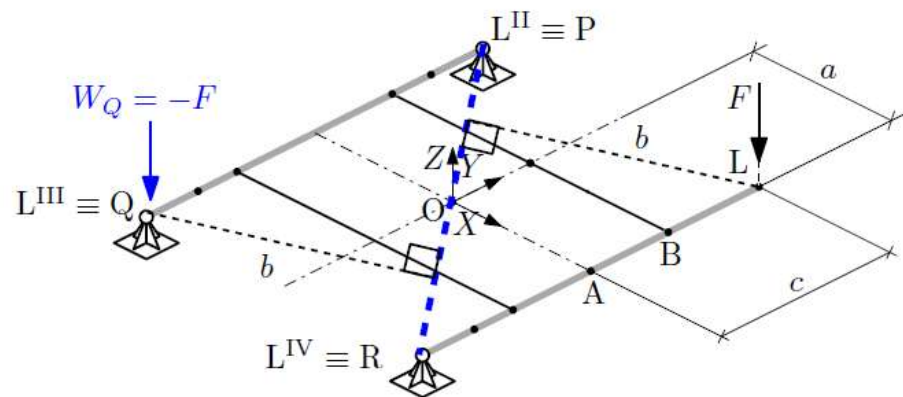
Castigliano's Theorem

CASE E: Ladder Frame – Nomenclature



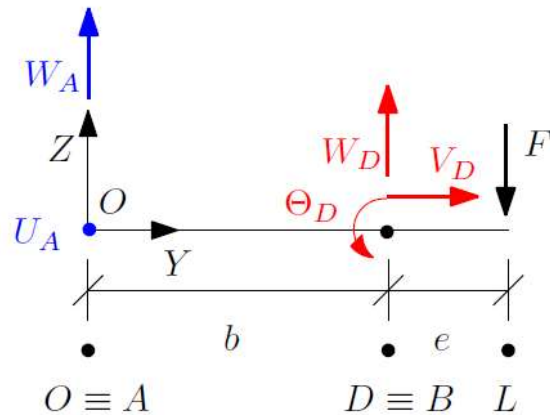
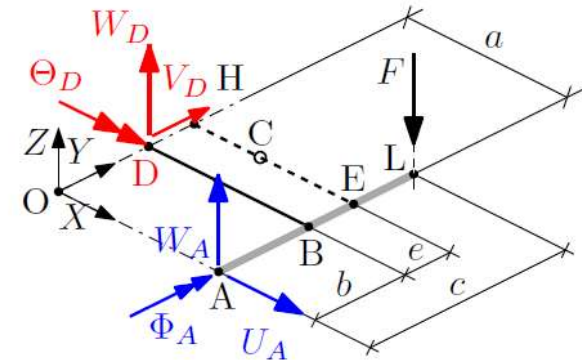
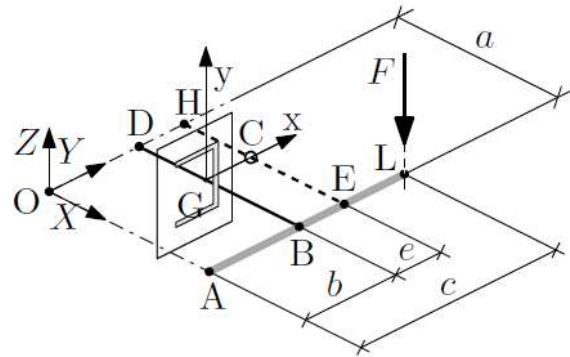
Ladder Frame

Equilibrium

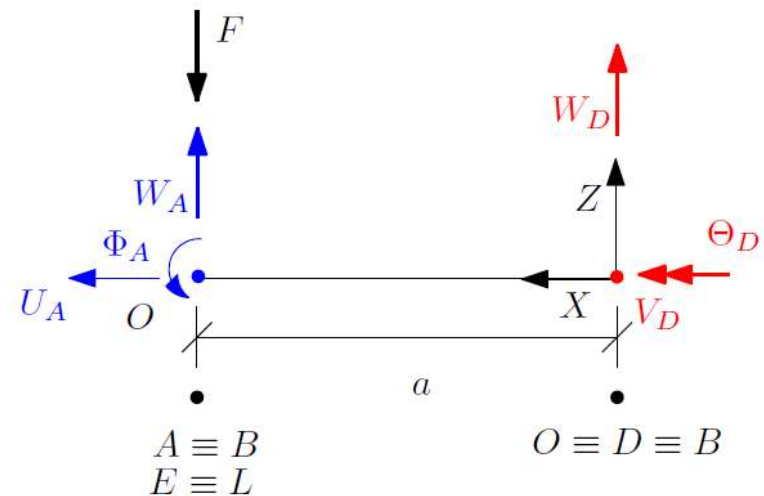


Ladder Frame

Screw-symmetry



X : outward



Y : outward

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References

LAB Maxima files saved as:

rollbar_def.wmx

quarter_ladder_frame_v005_BASE2019.wmx

“La vita è come andare in bicicletta. Per
mantener l’equilibrio devi muoverti”

A. Einstein



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