

# 37th SOLID MECHANICS CONFERENCE

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Book of Abstracts



POLISH ACADEMY OF SCIENCES  
Institute of Fundamental Technological Research  
and Committee on Mechanics

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*organized by*

Institute of Fundamental Technological Research (IPPT) and Committee on Mechanics  
POLISH ACADEMY OF SCIENCES

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**Preface**

This volume contains abstracts of papers accepted for presentation at the 37<sup>th</sup> Solid Mechanics Conference (SolMech 2010) held in Warsaw, September 6–10, 2010.

This 37<sup>th</sup> Conference belongs to a series of international Solid Mechanics Conferences (SolMech) continuing a long tradition going back to 1953. It is organized regularly, recently every two years, by the Institute of Fundamental Technological Research under the auspices of Committee on Mechanics, both of the Polish Academy of Sciences. Its aim is to bring together researchers from different countries to provide opportunities to exchange ideas, experiences and scientific research results from a wide area of solid mechanics. Over the years the Conference has evolved into an established event where world-wide leaders, their prospective successors and young researchers from Poland and abroad meet each other and share information on recent achievements and current trends in the mechanics of solids.

Following the tradition, Invited Plenary Lectures (7) constitute a distinguished part of the SolMech 2010 Conference. Keynote (14) and contributing (oral and poster) presentations have been organized into eleven Thematic Sessions as follows:

- Micromechanics, Interfaces and Multi-Scale Modelling (29 presentations)
- Fracture, Damage and Fatigue of Materials (23)
- Continuum Mechanics, Elasticity and Plasticity (25)
- Experimental Mechanics (in memory of Prof. W.K. Nowacki) (19)
- Biomechanics (9)
- Geomechanics (12)
- Smart Materials and Structures (8)
- Structural Mechanics and Optimization (8)
- Shells and Plates (19)
- Computational Aspects of Solid Mechanics (20)
- Nonlinear and Stochastic Dynamics (7).

On behalf of the Scientific and Organizing Committees of the 37<sup>th</sup> Solid Mechanics Conference (SolMech 2010) I wish all participants an inspiring and enjoyable time in Warsaw.

Warsaw, August 2010

Henryk Petryk

## NORMALIZATION OF THE STRESS CONCENTRATIONS AT THE INTERFERENCE FIT BETWEEN A CYLINDRICAL SHAFT AND A HUB WITH ROUNDED EDGES

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Interference fits are widely employed to connect gears, pulleys, flanges, wheels, and similar mechanical components, to a shaft. When a cylindrical shaft of infinite length is press-fitted into a cylindrical hub of finite length, stress concentrations take place at the hub-shaft contact extremities, whereas the contact stresses remain reasonably constant along the central portion of the contact, e.g. references [1-5]. If the edges of the hub bore are sharp, the elastic pressure peaks are mathematically infinite, whereas they become finite when the corners are rounded. To introduce fillets at the hub bore edges is therefore a practically relevant means for reducing the hub stress peaks. (Additional methods adopted to relieve the pressure peaks consist in introducing a shoulder in the shaft, and in adopting a grooved hub or shaft, see e.g. references [1,6,7].)

While the stress state at the hub sizeable central zone may be confidently evaluated by modelling the press-fit problem as plane and by employing the Lamè equations for thick-walled cylinders, e.g. references [1,7], the localised pressure peaks are not amenable to a simple analytical evaluation, e.g. references [2,5]. Consequently, the transmittable torque may be reliably estimated by resorting to the Lamè-based plane solution, that is valid along the majority of the contact axial length; conversely, a detailed, specific three-dimensional analysis is needed to evaluate the local stress peaks and to assess the strength of the hub.

In the following, it is assumed that the rounded edges of the hub are described by a quarter of circumference, so that the fillet radius fully describes the geometry of the mating surfaces at the hub-shaft contact extremities.

It is also noted that the contact pressure peaks depend on the shaft radius, the hub outer radius, the fillet radius, the interference, and the hub and shaft Young's moduli. Instead, the hub axial length is generally hardly relevant, provided that the hub is sufficiently long to produce, in its central part, a uniform stress state in the axial direction; this situation implies that the two pressure peaks at the contact extremities do not interact among each other. The normalised contact pressure peaks would depend on the previous radii normalised over a reference radius, but even so the number of variables seems to be too high to allow design charts to be compiled, that can cover a wide range of hub-shaft geometries and of materials adopted. In this paper a viable way to compile design charts addressing the stress peaks is presented for an infinitely long, solid cylindrical shaft, press-fitted into a cylindrical hub of finite axial thickness with rounded edges. In particular, it is shown in this study that, for a prescribed ratio between the hub inner and outer radii, the normalised stress peaks in the presence of radiused edges of the hub bore depend upon a parameter that comprises and summarises the simultaneous effects of a reference radius for the shaft-hub press-fit, the fillet radius, the interference, and the Young's modulus. For a prescribed hub radial aspect ratio, it is therefore now possible to prepare design charts in which the normalised von Mises stress peaks are diagrammatically presented versus the above normalising parameter.

It is assumed in this study that the hub-shaft contact is frictionless, since the coefficient of friction is low in this kind of applications, and the direction of the shear stresses acting at the hub-shaft contact in the axial direction is often undefined, since it depends on the assembly procedure. Consequently, fretting fatigue aspects, e.g. reference [8], are not examined in this study. The materials of the hub and shaft are assumed to exhibit the same Young's modulus. Finally, the contact problem is assumed as elastic, and yielding of the materials is not considered.

In this paper the above normalising parameter is determined by following the same approach successfully adopted in references [9,10], referring to a pin-in-plate contact problem, and to a press-

fit problem, respectively. To this aim, the contact problem is formally expressed in terms of an integral equation, where the kernel is normalised with respect to the Young's modulus and a reference radius. The known term of the integral equation represents the initial clearance and the overlapping between the two undeformed mating profiles of the hub and shaft. To normalise the integral equation, the initial clearance/overlapping must be approximated, as closely as possible, by a function formed by the product of a shape function by a representative variable. This is a key point of this approach, see e.g. references [9,11]. The integral equation may be normalised by considering a proper combination of the significant variables, and this approach allows the normalising parameter to be found.

The formulation of the title problem in terms of an integral equation serves to define the normalising parameter; however, since the integral equation is quite complex, e.g. references [2,5], the stress field is more conveniently evaluated by the finite element method, e.g. reference [12], by varying the normalising parameter within an interval covering practically significant shaft-hub press-fit configurations, e.g. reference [13]. The introduction of a normalising parameter allows the stress forecasts to be presented in way that is both more compact and of ample validity, with respect to papers analysing specific problems, e.g. references [14,15].

In this paper, preliminary design diagrams are presented that express the normalised stress concentration factor in the hub versus the above normalising parameter for a selection of hub radial aspect ratios. Such diagrams assist the designer in selecting the correct radius for the rounded edge of the hub, or in assessing the hub stress level in the shaft-hub press-fit.

### References

- [1] Kanber, B., Boundary element analysis of interference fits. *Turkish J. Eng. Env. Sci.*, 30, 323-330, 2006.
- [2] Goodier, J.N., Loutzenheiser, C.B. Pressure peaks at the ends of plane strain rigid die contacts (elastic). *J. Appl. Mech.* 32, 462-463, 1965.
- [3] Conway, H.D., Farnham, K.A. Contact stresses between cylindrical shafts and sleeves. *Int. J. Engng Sci.*, 5, 541-554, 1967.
- [4] Strozzi, A. Static stresses in an unpressurized, rounded, rectangular, elastomeric seal. *ASLE Transactions*, 29, 558-564, 1986.
- [5] Ciavarella, M., Hills, D.A., Monno, G. The influence of rounded edges on indentation by a flat punch. *Proc. Instn Mech. Engrs*, part C, 212, 319-328, 1998.
- [6] Bijak-Zochowski, M., Marek, P., Tracz, M. On methods of reduction and elimination of stress singularities in some elastic contact problems. *Int. J. Mech. Sci.*, 36, 279-296, 1994.
- [7] Strozzi, A. *Costruzione di Macchine*, 1998 (Pitagora, Bologna).
- [8] Lanoue, F., Vadean, A., Sanschagrin, B., Finite element analysis and contact modelling considerations of interference fits for fretting fatigue strength calculations. *Simulation modelling practice and theory*, 17, 1587-1602, 2009.
- [9] Ciavarella, M., Baldini, A., Barber, J.R., and Strozzi, A. Reduced dependence on loading parameters in almost conforming contacts. *Int. J. Mech. Sci.*, 2006, 48, 917-925.
- [10] Strozzi, A., Baldini, A., Giacomini, M., Rosi, R., Bertocchi, E. Contact stresses within a split ring inserted into a circular housing. *J. Strain Analysis*, 44, 2041-3130, 2009.
- [11] Schmelz, F., Seher-Thoss, H.C., Aucktor, E. *Universal Joints and Driveshafts*. Springer-Verlag, Berlin, 1992. (English version).
- [12] White, D.J., Humpherson, J., Finite-element analysis of stresses in shafts due to interference-fit hubs. *J. Strain Analysis*, 4, 105-114, 1969.
- [13] Castagnetti, D., Dragoni, E., Optimal aspect ratio of interference fits for maximum load transfer capacity. *J. Strain Analysis*, 40, 177-184, 2005.
- [14] Hang, Y., McClain, B., Fang, X.D., Design of interference fits via finite element method. *Int. J. Mech. Sci.*, 42, 1835-1850, 2000.
- [15] Dobromirski, J., Smith, I.O., A stress analysis of a shaft with a press-fitted hub subjected to cyclic axial loading. *Int. J. Mech. Sci.*, 28, 41-52, 1986.