TORQUE TRANSMISSION BY FRICTION IN A KEYED SHAFT-HUB PRESS-FITS

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ABSTRACT. The transmissible torque in keyed shaft-hub press-fits is examined. In a shaft-hub press-fit, the presence of a keyseat increases the compliance of both shaft and hub, thus reducing the contact pressure and, therefore, the torque transmitted by friction with respect to a keyless coupling. Such torque diminution is explored with Finite Elements for a solid shaft, for a selection of hub aspect ratios, and for various practically relevant keyseat geometries. Various design diagrams of prompt access are compiled.

1 INTRODUCTION

Interference fits are widely employed to semi-permanently connect gears, pulleys, flanges, wheels, disks, rotors, and similar mechanical components, to a shaft. When a cylindrical shaft of infinite length is press-fitted into a keyless cylindrical hub of finite axial length, the contact pressure is axisymmetrically distributed; stress concentrations take place at the hub-shaft contact extremities, whereas the contact stresses remain reasonably constant in the shaft axial direction along a sizeable central portion of the contact, *e.g.* reference [1]. The axisymmetric stress state along the hub central portion may be thoroughly predicted by modelling the press-fit problem as plane and axisymmetric, and by employing the Lamé equations for thick-walled cylinders; the torque transmitted by friction may be confidently estimated by relying on the Lamé predictions, since they are valid along most of the contact axial length, *e.g.* reference [1].

Often a key is added to the press-fit, to secure the torque transmission and to lock the shaft and the hub in a definite angular position. Both parallel and tapered keys are employed. When a parallel key is used, a reduced interference with respect to its keyless counterpart is usually adopted in the shaft-hub press-fit, since the key provides a back-up means for torque transmission. Instead, when a tapered key is employed, a precision fit with centring purposes is usually adopted between shaft and hub, and the outcome of the frictional forces necessary to transmit torque is committed to the radial compression exerted by the tapered key. As tapered keys drive all the radial clearance to one side, they tend to create eccentricity between hub and shaft, a drawback that is not encountered with parallel keys.

This study concerns parallel keys only. The following notes summarize the recommended shapes and dimensions of the key cross section and of the keyseat. Both square and rectangular key cross sections are adopted in practice, where square keys are recommended for shafts of diameters up to 25 mm, whereas rectangular keys are adopted for larger diameters, *e.g.* reference [2], p. 562. The suggested key width, *w*, remains constant within prescribed intervals of the shaft diameter. Two different key widths are recommended in the standards: a) according to the ANSI B17.1 standards, the key width is about 0.25 times the mean shaft diameter for a general interval; b) according to the BS 4235, ISO 2491, DIN 6885 standards, the key width is about 0.3 times the mean shaft diameter.

In the square keys, the key height, *h*, is equal to its width, *w*. For rectangular keys, the recommended height varies considerably with the standards; in particular, for shaft diameters ranging from 25 to 50 mm, the height of the rectangular key varies from about 0.4 to 0.8 times its width.

The keyseats in the shaft and the hub are normally designed so that exactly one-half of the height of the key is bearing on the side of the shaft keyseat, and the other half on the side of the hub keyseat, *e.g.* reference [2], p. 562. Rectangular keys cut deeper into the shaft than they are cut into the hub are also employed.

Concerning the tolerances, an interference or a clearance may occur between the key sides and the keyseat lateral walls.

With respect to a keyless press-fit, the presence of the keyseat increases the radial compliance of both the shaft and the hub, and, therefore, it reduces the shaft-hub mean contact pressure and the torque transmitted by friction. A second aspect causing a diminution of the transmissible torque is that, as a result of the presence of the keyway, the contact pressure acts along an arc of reduced length with respect to the whole shaft periphery.

It is underlined that the transmissible torque in the presence of the keyseat is defined in this paper as the torque transmitted by the effect of the contact pressure between shaft and hub, from which the limit interface shear stress distribution may be predicted. In other words, the transmissible torque is the torque transmitted by friction alone. In fact, the motivation of this paper is to assist the designer is dimensioning a press-fit in which the key plays a secondary, safety role in the torque transmission. Conversely, the torque directly transmitted by the key, which becomes predominant when slippage occurs between shaft and hub, is not examined in this paper.

In this paper a plane, non axisymmetric Finite Element analysis of the shaft-hub contact is carried out in the presence of a keyseat to quantify the above diminution of the mean contact pressure and of the transmissible torque. (The stress concentrations induced by the presence of the keyseat are outside the scope of this paper.) The analysis is carried out for a solid shaft, for a selection of hub aspect ratios and for recommended keyseat geometries. Several design diagrams are compiled, that allow the diminution of the transmissible torque to be estimated for a selection of hub aspect ratios.

2. SIMPLIFYING ASSUMPTIONS

The contact pressure between shaft and hub produces a circumferential contraction (expansion) of the shaft (hub) keyseat, see Figure 1. Both such distortions increase the radial compliance of the hub and the shaft, and, therefore, they reduce the shaft-hub mean contact pressure with respect to a keyless press-fit.

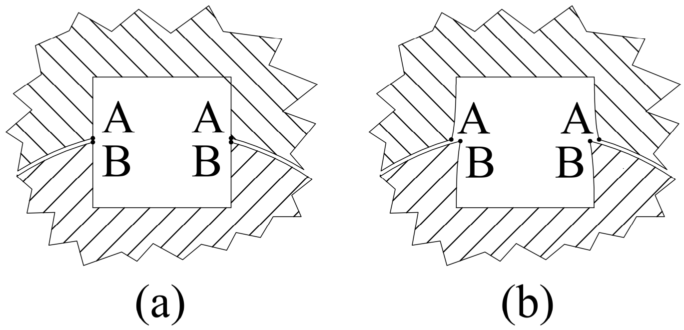

FIGURE 1. (a) Keyseat undeformed shape; (b) Keyseat deformed shape

For a keyed connection whose hub exhibits a prescribed axial length, the torque transmitted by friction may be estimated by noting that the central part of the shaft-hub contact is representative of most of the contact axial length. This central part may be modelled as a plane, non axisymmetric problem, in which the presence of the keyseat in both the shaft and the hub is accounted for. With this assumption, a three-dimensional analysis of the title problem becomes unnecessary, and a considerably simpler two-dimensional model may suffice.

Concerning the tolerances between the key lateral walls and the keyseat sides, from the indications traceable in the pertinent literature it may be deduced that an interference or a clearance may occur. The presence of a clearance facilitates key insertion and removal. However, in the case of alternating torque, any clearance between key and keyseat would be suddenly taken up, with resulting impact and undesired high stresses, *i.e.* with backlash, reference [2], p. 563. In such cases, interference is highly recommended.

The maximum diminution of the transmissible torque is attained when the maximum increase is achieved of the shaft and hub compliance in the radial direction, as a result of the presence of the keyseat. This condition is attained when the shaft (hub) keyseat may freely contract (expand) circumferentially under the effect of the shaft-hub press-fit pressure. Independent of the presence of interference or clearance between the keyseat and the key lateral walls, the expansion of the hub keyseat is not precluded by the presence of the key. Instead, the degree of contraction of the shaft keyseat noticeably depends upon whether an initial clearance or an interference occurs between the key sides and the lateral walls of the shaft keyseat. In conclusion, the diminution in the transmissible torque depends on the tolerances adopted for the key and the keyseat. Since the aim of this paper is to evaluate the maximum possible diminution of the transmissible torque, it was decided to consider the extremal reference situation of an initial clearance between the key sides and the lateral walls of the shaft keyseat, whose value is sufficient to guarantee that the shaft keyseat contracts freely. In other words, in the plane modelling of the shaft-hub press-fit favoured in this paper, the shaft and hub keyseats are carefully modelled, but the presence of the key is neglected. With this assumption, the forecast torque diminution is extremal and independent of the clearance or interference adopted between key and keyseat.

The shaft-hub contact has been assumed as frictionless. Since in most frictional contacts the contact pressure is relatively independent of the coefficient of friction, *e.g.* reference [3], a plausible simplifying assumption is to evaluate the shaft-hub contact pressure in the absence of friction, and to estimate the transmissible torque by computing the limit shaft-hub interface shear stress distribution as the product of the contact pressure by the coefficient of friction.

3. FINITE ELEMENT ANALYSIS

The commercial finite element program MSC Marc 2010 has been employed in this study. A fine mesh has been adopted, of about 10000 nodes. Plane stress has been assumed, consistent with the Lamé analytical solution. The shaft has been modelled as solid. A selection of hub inner, r_i, to outer, r_o, radii ratios comprised between 0.3 and 0.7 has been considered, as in reference [1]. A keyed hub with small radial thickness, in the region of $r_i/r_o = 0.7$, has been examined in reference [4].

The shaft-hub contact has been considered as frictionless. The shaft and hub materials have been assumed as elastic, with a Young's modulus of 210000 MPa and a Poisson's ratio of 0.3, typical of steel.

Since the keyseat deforms as illustrated in Figure 1 (b), the edges of the hub keyseat, lettered *A*, indent the shaft surface, thus causing localized infinite elastic pressure peaks. Conversely, if reference is made to the undeformed geometry, Figure 1 (a), each edge of the hub keyseat, lettered *A*, is perfectly aligned with the corresponding edge of the shaft keyseat, lettered *B*. Contrary to its deformed counterpart, in this undeformed configuration the contact pressure remains finite in the vicinity of the edges, see Figure 9 (a) of reference [5]. Although the deformed configuration depicted in Figure 1 (b) thoroughly describes the actual keyseat distortion, reference has been made to the undeformed geometry of Figure 1 (a) for the following reasons: a) the pressure peaks occurring in the configuration of Figure 1 (b) are so localized, that they do not appreciably affect the value of the transmissible torque, which depends on the mean value of the contact pressure and not on local features; b) the Finite Element contact pressure is not affected by unphysical oscillations, often appearing in the zones of high stress gradients; c) the angular distance between the edge of the hub keyseat, lettered *A* in Figure 1 (b), and the corresponding edge of the shaft keyseat, lettered *B* in Figure 1 (b) defines the arc of the shaft surface that is pressure-free. Since the extent of this unloaded arc varies with the press-fit interference, this contact problem modelled in the deformed configuration of Figure 1 (b) is (moderately) nonlinear with the interference imposed, whereas the undeformed configuration of Figure 1 (a) describes a linear, stationary contact problem, *e.g.* reference [6], which may be rigorously normalized; d) it is planned to develop a Michell-type analytical solution of the title problem, *e.g.* reference [7], for which infinite pressure peaks would be particularly difficult to mimic. In Section 5 it will be shown that the two above approaches supply very similar forecasts in terms of transmissible torque.

4. RESULTS

Figure 2 displays a typical FE output for a keyed connection defined by $r_i/r_o = 0.5$ and for a square key defined by $w/d_i = 0.3$. The contact pressure, p, is uniform in the zones far away from the keyseat, it exhibits a local maximum in the vicinity of the key, and then it diminishes, remaining finite at the keyseat edge.

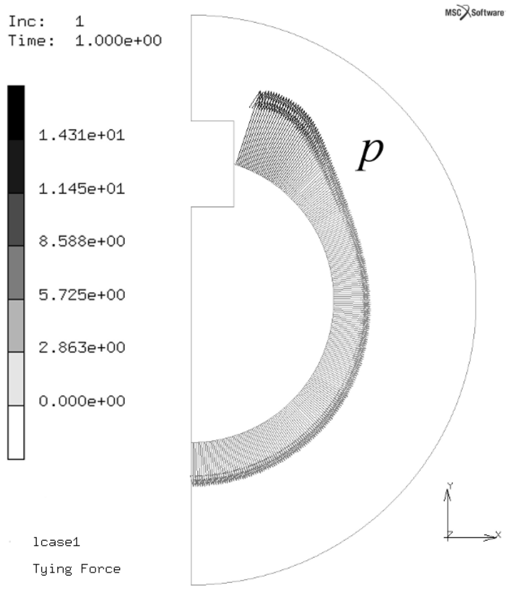

FIGURE 2. Distribution of the contact pressure, *p*, for and for a square key defined by r_i/r_o = 0.5 and w/d_i = 0.3

The local maximum is activated by the need to compensate for the lack of contact pressure in the keyed zone in the vertical equilibrium equation, see Figure 4 of reference [8] for a similar aspect. The diminution in contact pressure in the vicinity of the keyseat is confidently attributable to the contact problem being similar to a plane strain situation in the zones sufficiently remote from the keyseat, and to a more deformable plane stress at the keyseat edge, *e.g.* reference [9], p. 132. According to reference [9], the contact pressure at the keyseat edge is 1-υ2 times the local maximum; this forecast reasonably agrees with the FE pressure reduction in the region of 10 per cent for υ=0.3.

Figure 3 addresses parallel square keys only, and it reports the reduction of the transmissible torque caused by the presence of the keyseat. The shaft is assumed as solid. The following selection of hub inner, r_i, to outer, r_o, radii ratios has been considered, namely r_i/r_o = 0.3, 0.4, 0.5, 0.6, 0.7, as in reference [1]. The aspect ratio r_i/r_o has been reported along the *x*-axis, whereas the ratio between the transmissible torque in the presence of a keyseat, T_{key}, and its analogue for a keyless coupling, *T*, has been reported along the *y*-axis. Two curves are presented, referring to two ratios of the square key width, *w*, to the shaft diameter, d_i, namely w/d_i = 0.25 and 0.3, see the Introduction for details.

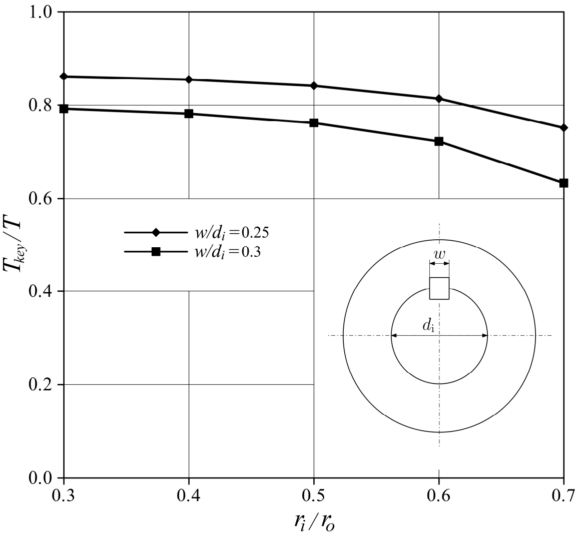

FIGURE 3. T_{key} / T versus r_i/r_o for two square keys defined by $w/d_i = 0.25$ and 0.3

It is noted that both T and T_{key} depend linearly on the coefficient of friction f; consequently T_{key}/T is independent of f. This is a positive occurrence, since f is particularly difficult to estimate, *e.g.* reference [1].

As noted in the Introduction, the standards require that a constant value of the key width be adopted within prescribed intervals of the shaft diameter. The corresponding diagram expressing the reduction of the transmissible torque would therefore be formed by a sequence of curves that are discontinuous at the transition points between two adjacent intervals of the shaft diameter. In the interest of simplicity, it was decided to adopt a procedure leading to a continuous curve, by referring to a keyseat width equal to 0.25 and 0.3 times the local value of the shaft diameter, and not to the mean value of the diameter for each considered interval of the shaft diameter.

The diminution of the transmissible torque becomes appreciable when the radial thickness of the hub is relatively small, that is, for high values of the r_i/r_o aspect ratio, and when the keyseat width is relatively high; for $r_i/r_o = 0.7$, the ratios between the transmissible torque in the presence of a keyseat and its keyless counterpart are 0.633 and 0.751 for keyseat widths equal to 0.3 and 0.25 times the local value of the shaft diameter, respectively.

Figures 4 to 6 address parallel rectangular keys. As for square keys, two key widths, w, are considered, namely 0.25 and 0.3 times the shaft diameter d_i. In addition, three values of the key height, h, are addressed, namely 0.4, 0.6, and 0.8 times the key width w. The height of the keyseat lateral walls is assumed to be equal in the shaft and in the hub, see the Introduction for details.

Figures 4 to 6 address the two values $w/d_i = 0.25$ and 0.3; Figure 4 considers h/w=0.4, Figure 5 addresses $h/w = 0.6$, and Figure 6 deals with $h/w = 0.8$. The diminution in transmissible torque is lower than that encountered with square keys; the minimum value of the ratio between the transmissible torque in the presence of a keyseat and its keyless counterpart is about 0.721 and occurs for $h/w = 0.8$ and for $r_i/r_o = 0.7$.

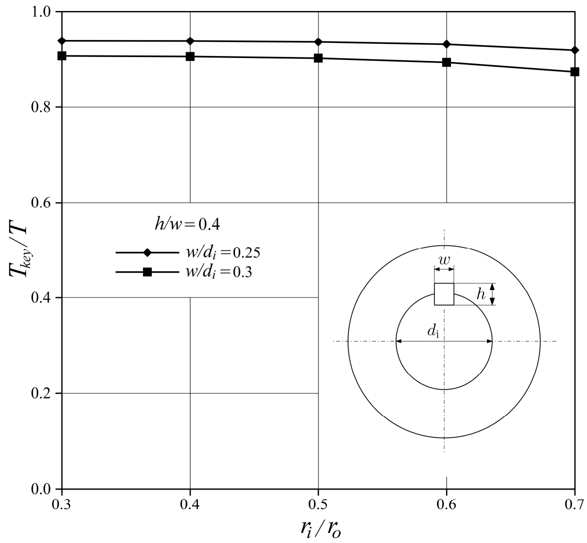

FIGURE 4. T_{key} / T versus r_i/r_o for two rectangular keys defined by $w/d_i = 0.25$ and 0.3, and by $h/w = 0.4$

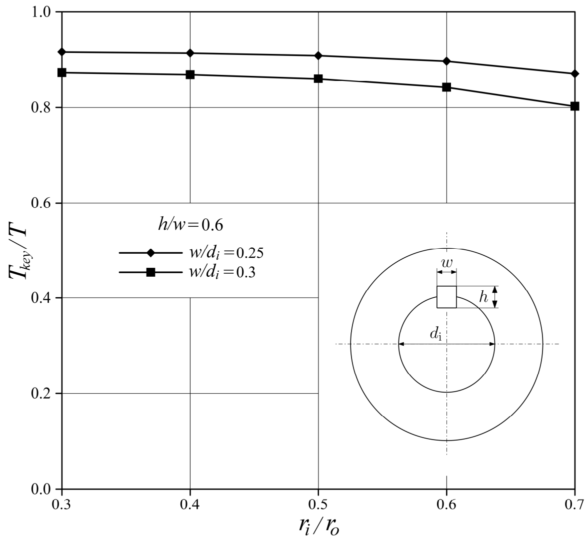

FIGURE 5. T_{key} / T versus r_i/r_o for two rectangular keys defined by $w/d_i = 0.25$ and 0.3, and by $h/w = 0.6$

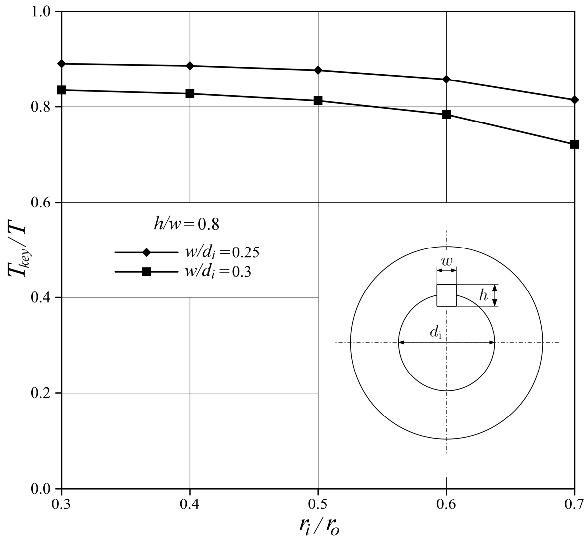

FIGURE 6. T_{key} / T versus r_i/r_o for two rectangular keys defined by $w/d_i = 0.25$ and 0.3, and by h/w = 0.8

It may be conjectured that, when the hub radial thickness becomes very high, the presence of the keyseat produces a localized perturbation of the stress field with respect to an axisymmetric configuration and, therefore, the ratio between the transmissible torque in the presence of a keyseat and its keyless counterpart may be expected to approach unity. Instead, the diagrams of Figures 3 to 6 clarify that the above ratio approaches a value appreciably lower than unity for vanishing r_i/r_o aspect ratios. This trend may be rationalized by observing that, even when the hub radial thickness becomes very large, the contact pressure in the presence of a keyseat does not act along a whole circumference, since it is reduced by the presence of the keyseat. With reference to Figure 3, for $w/d_i = 0.3$ the (almost asymptotic) y-value is 0.792, whereas for $w/d_i = 0.25$ it becomes 0.861. The ratio between the arc subjected to the contact pressure and the whole shaft periphery is 0.95 for w/d_i=0.3 and 0.96 for $w/d_i = 0.25$. Consequently, even for high values of the hub radial thickness, the diminution of the transmissible torque is partially imputable to a decay of the mean contact pressure, and partly to the reduction of the contact arc. It is difficult to compute the analytical asymptotic value of the above ratio for r_i/r_o approaching zero.

5. ASSESSMENT OF SOME SIMPLIFYING ASSUMPTIONS

An alternative, extremal assumption with respect to the frictionless contact between shaft and hub is to adopt an infinite coefficient of friction, that is, to assume perfect adhesion between the mating surfaces. An assessment of the influence of the coefficient of friction on the transmissible torque reduction has been carried out for a situation in which the torque reduction is particularly appreciable, namely for a parallel square key, for an hub aspect ratio $r_i/r_o = 0.7$, and for $w/d_i = 0.3$. The ratio between the transmissible torque for null friction (with reference to the undeformed keyseat of Figure 1 (a)) and for infinite friction is 1.044. This result shows that the influence of friction on the integral of the contact pressure is negligible.

In Section 2 the keyseat distortion represented in Figure 1 (b) has been discussed. The forecasts of Section 4 have been retrieved by referring to the undeformed keyseat represented in Figure 1 (a). A test case was run to assess the influence of the keyseat deformed/undeformed profile on the transmissible torque. The contact has been assumed as frictionless. The geometry considered is that of the previous assessments. In this case, however, it is necessary to refer to specific values of the shaft diameter and of the diametral interference, since this contact problem is (moderately) nonlinear, see Section 3. Reference has been made to $d_i = 10$ mm and to a diametral interference of 0.02 mm, see reference [1]. The ratio between the transmissible torque referring to the undeformed and to the deformed keyseat is 1.001. This result fully justifies the assumption made in Section 3, according to which the transmissible torque may be evaluated by referring to the undeformed configuration of Figure 1 (a).

In Section 2 it has been noted that the distortion of the shaft keyseat may be precluded by the presence of the key when the key-keyseat coupling is a precision fit. It may additionally be observed that in axisymmetric couplings the solid shaft is generally much stiffer than the hub, *e.g.* reference [10], p. 698. It may therefore be argued that in practical circumstances the increase in the shaft-hub radial compliance is attributable to the presence of the hub keyseat more than of the shaft keyseat. To assess the relative importance of the shaft and hub keyseats, a shaft-hub press-fit has been considered in which the shaft is solid and perfectly axisymmetric, and only the hub keyseat is present. The contact has been assumed as frictionless. The geometry considered is that of the previous assessments. The ratio between the transmissible torque in the presence of both keyseats and the torque in the presence of the hub keyseat but in the absence of the shaft keyseat is 0.872. This ratio is not sufficiently close to unity to justify the idealization relying on neglecting the presence of the shaft keyseat.

6. CONCLUSION

The transmissible torque in keyless and keyed shaft-hub press-fits has been examined. In a shaft-hub press-fit, the presence of a keyseat increases the compliance of both shaft and hub, thus reducing the contact pressure and, therefore, the torque transmitted by friction with respect to a keyless coupling. Such torque diminution has been explored with Finite Elements for a solid shaft, for a selection of hub aspect ratios, and for various practically relevant keyseat geometries. Several technically significant design diagrams have been compiled. For a hub inner to outer radii ratio of 0.7, the torque transmitted by friction in the presence of a square key is about 63 per cent of its keyless counterpart.

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