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Preliminary Analysis Of The Influence Of Textured Surfaces On The Fluid Film Behavior In Hip Replacements Via A Mass-Conserving Complementarity Algorithm

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ABSTRACT

The tribological behavior of hip prostheses is a very important factor for the success of the implants. The lubrication of the coupling between the head and the cup mainly influences both the contact problem behavior and the wear of the prosthesis. In particular, wear is recognized to play a crucial role in the implant failure. Many theoretical and numerical studies are available on the lubrication behavior of hip joint replacements; this has spurred the development of different procedures that allow the computation of both the lubricant film thickness and the pressure profile, under different loading conditions. As confirmed by many authors, the lubrication regime governing a hip joint is mainly elastohydrodynamic. However, only few studies are available in which cavitation phenomena have been accounted for in the investigation of the interaction between the two mating surfaces. Moreover, cavitation is usually implemented employing the simplified Reynolds condition at the rupture boundary, an approximation that may cause a lack of conservation of the mass. It is well known that textured surfaces can generally increase the load capacity in bearings, thus improving the lubrication effects. Unfortunately, very few studies have investigated the application of texture patterns to hip couplings. To the best authors' knowledge, no studies are available that investigate this behavior using a mass-conserving code. This contribution aims at investigating the behavior of a textured hip joint coupling employing a mass-conserving complementary code in order to investigate the potential benefit that the micro-cavitation could have on the global behavior of the joint under a certain history load, as a result of the textured pocket. It is planned to account for both squeeze and sliding effects.

Raynolds equation for a compressible fluid in one dimension

$$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{6\mu}\frac{\partial p}{\partial x}\right)-U\frac{\partial(\rho h)}{\partial x}-2\frac{\partial(\rho h)}{\partial t}=0$$

Considering :

- The density to be indipendent of pressure (i.e. density affected only by cavitation)
- Therefore density, ρ(x,t), is always less than or equal to the fluid density ρ_0

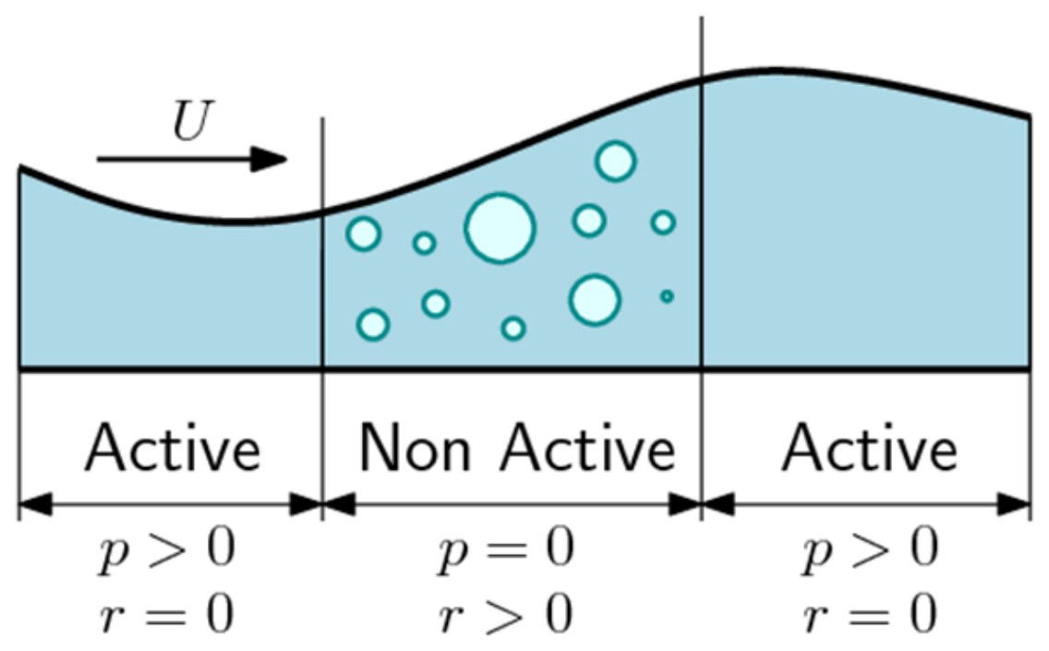
It is possible to define complementarity variables

$$r=1-\frac{\rho}{\rho_0}$$

$p \geq 0$ Pressure

$r \geq 0$ Void Fraction

$p \cdot r = 0$



The hydrodynamic lubrication can be solved as a Linear Complementarity Problem with the formulation

$$\begin{cases}\frac{\partial}{\partial x}\left(\frac{h^3}{6\mu}\frac{\partial p}{\partial x}\right)-U\frac{\partial h}{\partial x}-2\frac{\partial h}{\partial t}+U\frac{\partial(rh)}{\partial x}+2\frac{\partial(rh)}{\partial t}=0\\ p\geq 0;\quad r\geq 0;\quad p\cdot r=0\end{cases}$$

This LCP formulation of Reynolds problem is solved with the Galerking method whose numerical formulation is

$$F=\int_{\Omega}W\left(\frac{\partial}{\partial x}\left(\frac{h^3}{6\mu}\frac{\partial p}{\partial x}\right)-U\frac{\partial h}{\partial x}-2\frac{\partial h}{\partial t}+U\frac{\partial(rh)}{\partial x}+2\frac{\partial(rh)}{\partial t}\right)\Delta\Omega$$

After insertion of shape functions

$$F=[A]p+[B]r+C=0$$

where

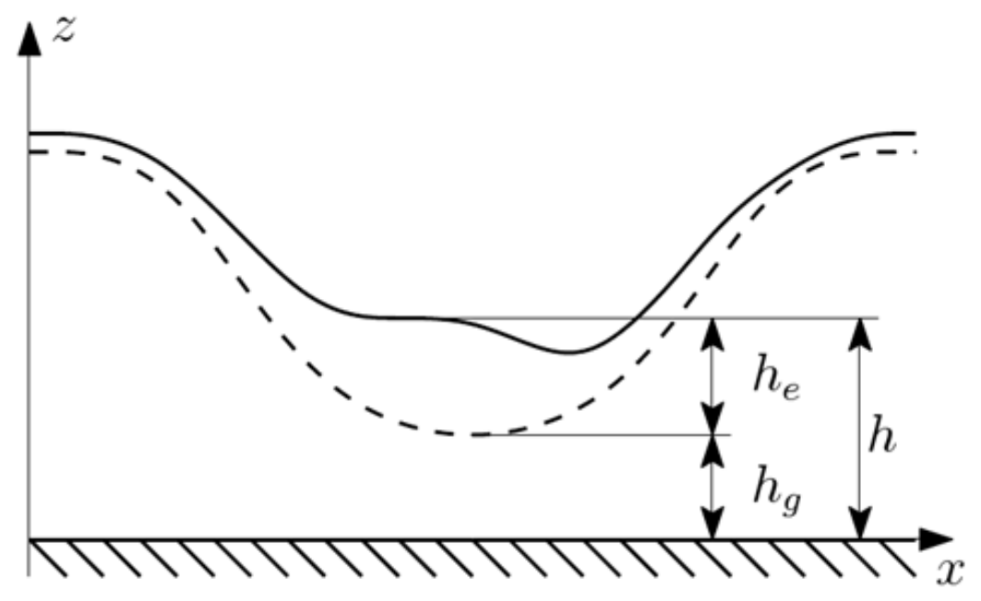
$$A_{j,k}=-\sum_{n=1}^{N_e}\frac{1}{6\mu}\sum_{m=1}^{N_{gp}}\left(h_m^3\sum_{k=1}^{N_n}\frac{\partial N_{mk}}{\partial x}\frac{\partial W_{mj}}{\partial x}\right)\Delta\Omega_m$$

$$B_{j,k}=\sum_{n=1}^{N_e}\sum_{m=1}^{N_{gp}}\left(U\sum_{k=1}^{N_n}W_{mj}h_m\frac{\partial N_{mk}}{\partial x}+2\frac{1}{\Delta t}\sum_{k=1}^{N_n}W_{mj}N_{mk}h_m(t)\right)\Delta\Omega_m$$

$$C_{j,k}=\sum_{n=1}^{N_e}\sum_{m=1}^{N_{gp}}\left(UW_{mj}\frac{\partial h_m}{\partial x}+2W_{mj}\frac{h_m(t)-h_m(t-\Delta t)}{\Delta t}+2\frac{1}{\Delta t}\sum_{k=1}^{N_n}W_{mj}N_{mk}r_k(t-\Delta t)h_m(t-\Delta t)\right)\Delta\Omega_m$$

The elastic problem is considered by coupling the Raynolds equations with the solution of elastic problem

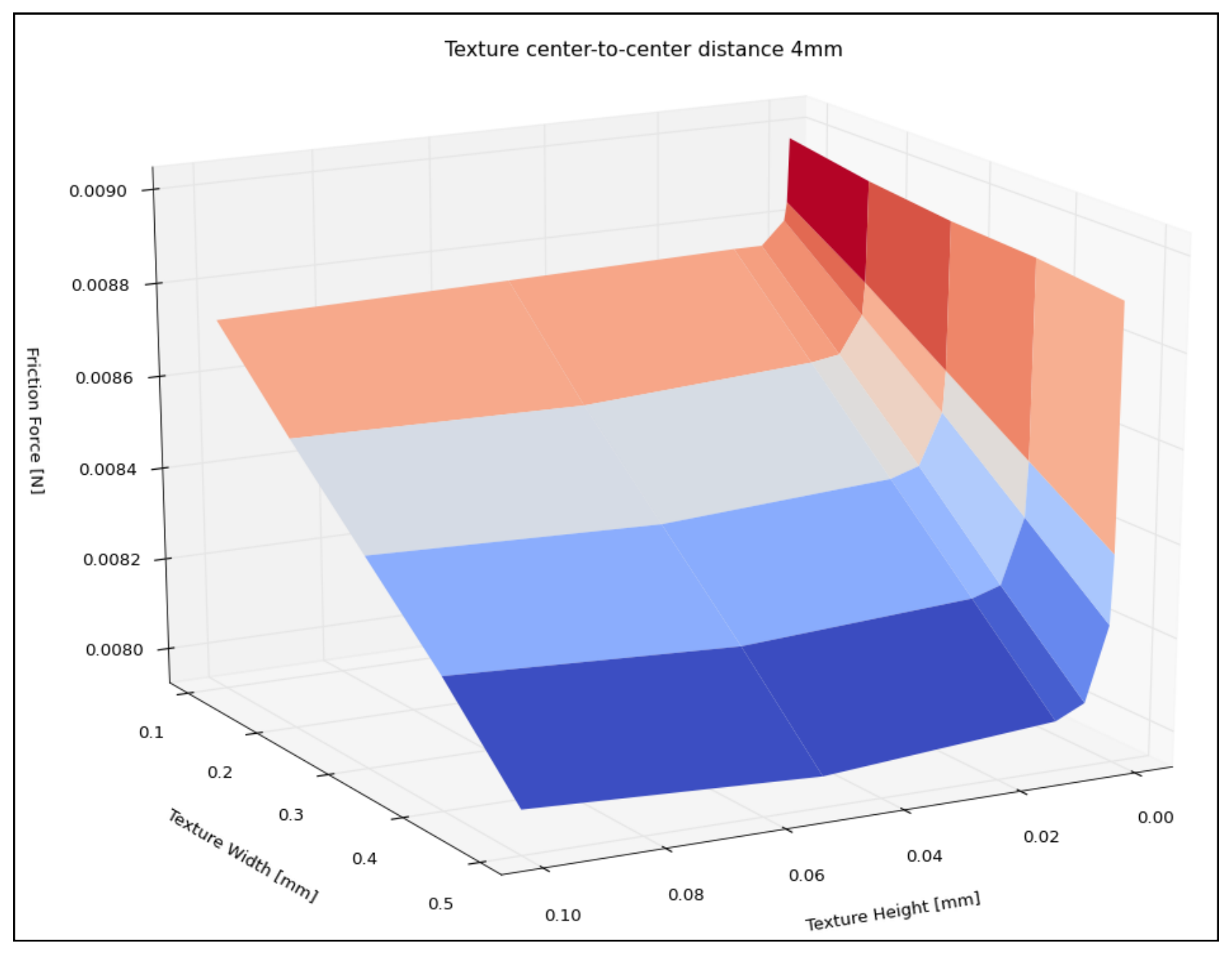
$$h=h_g+h_e$$
$$h_e=[C]\cdot p$$



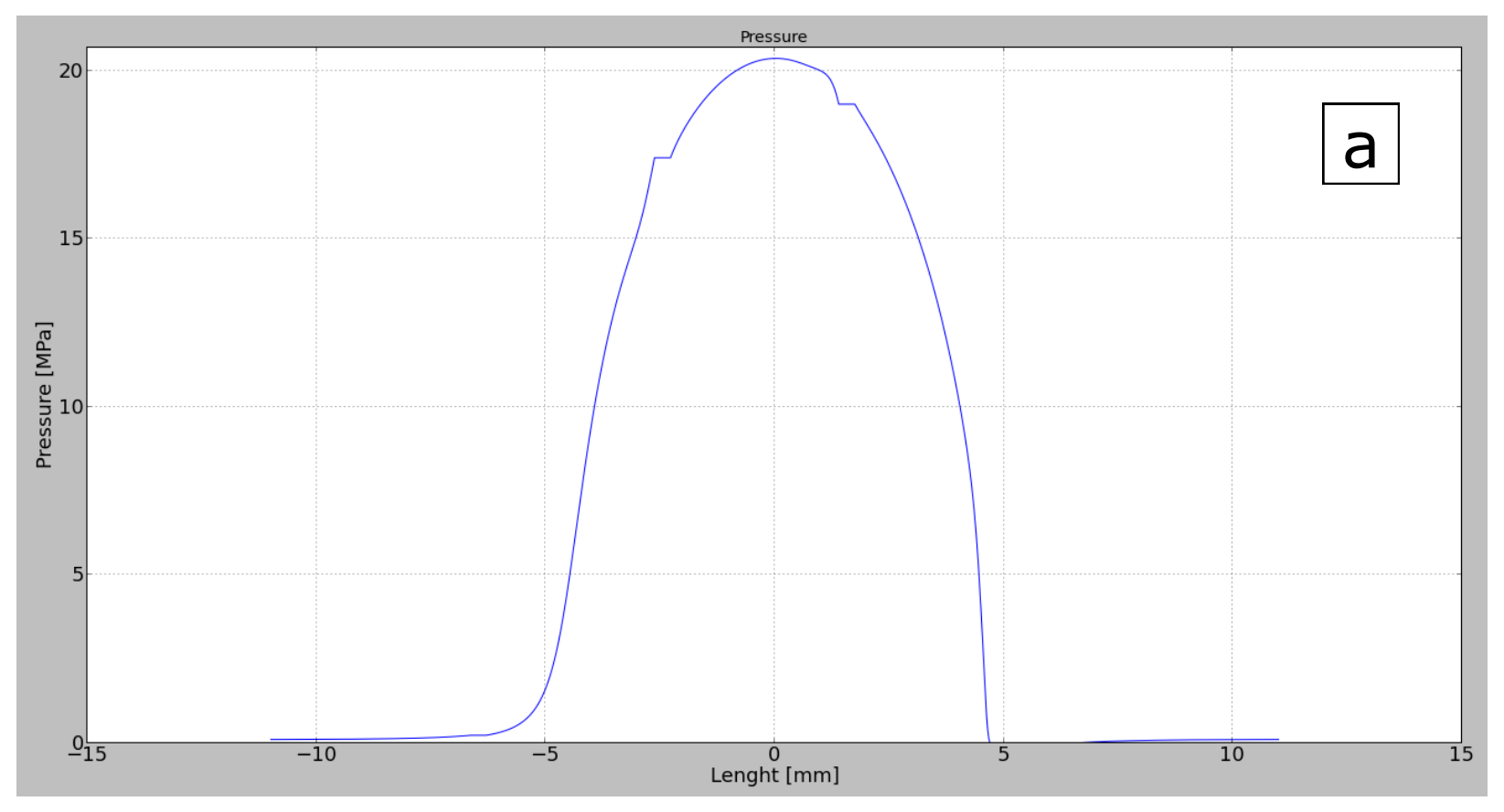
The problem is strongly non linear, so the initial formulation is modified considering the functional derivative of the operator F.

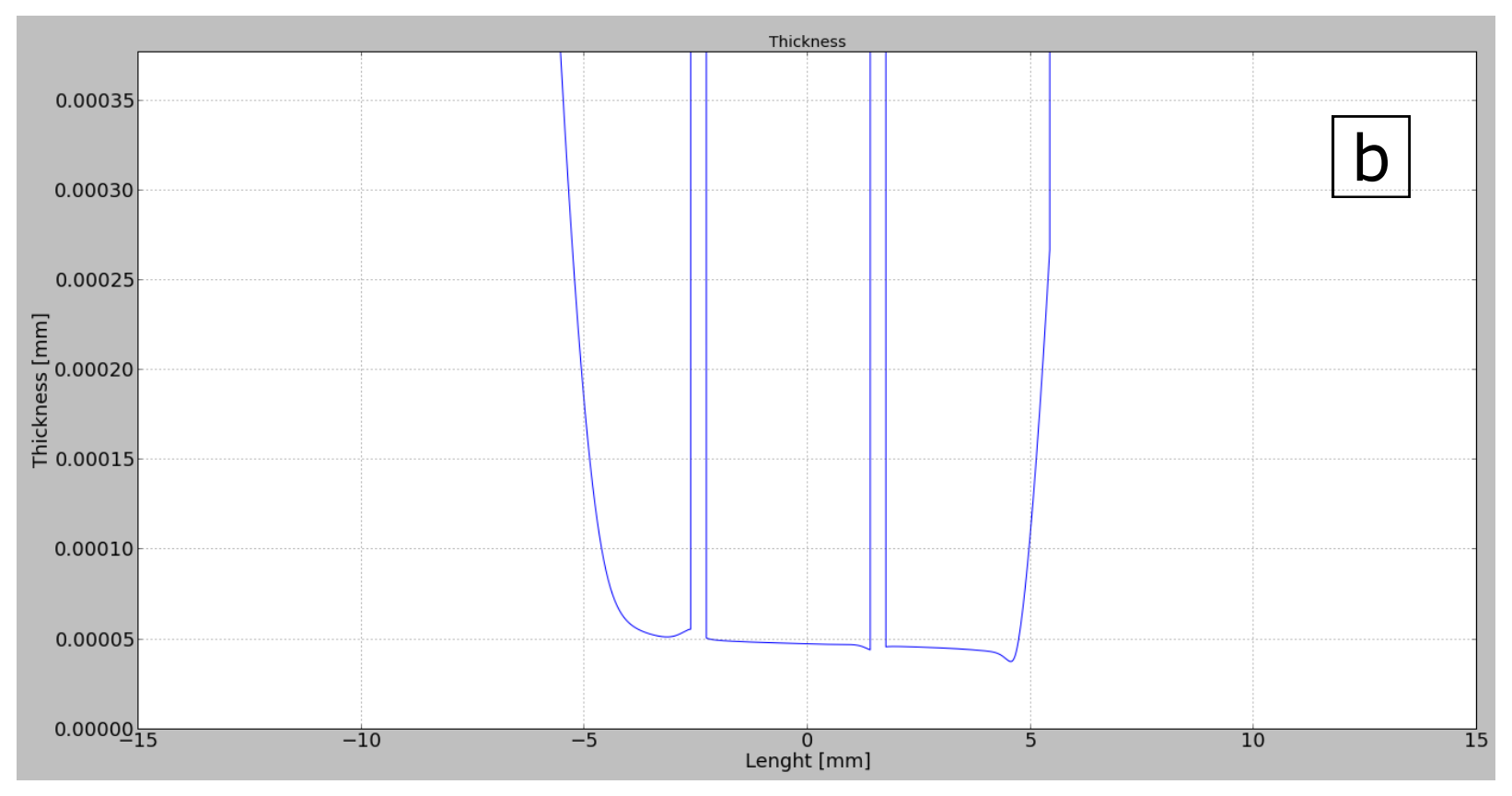
Moreover the mixed lubrication condition is considered introducing a dry contact problem solved as a LCP

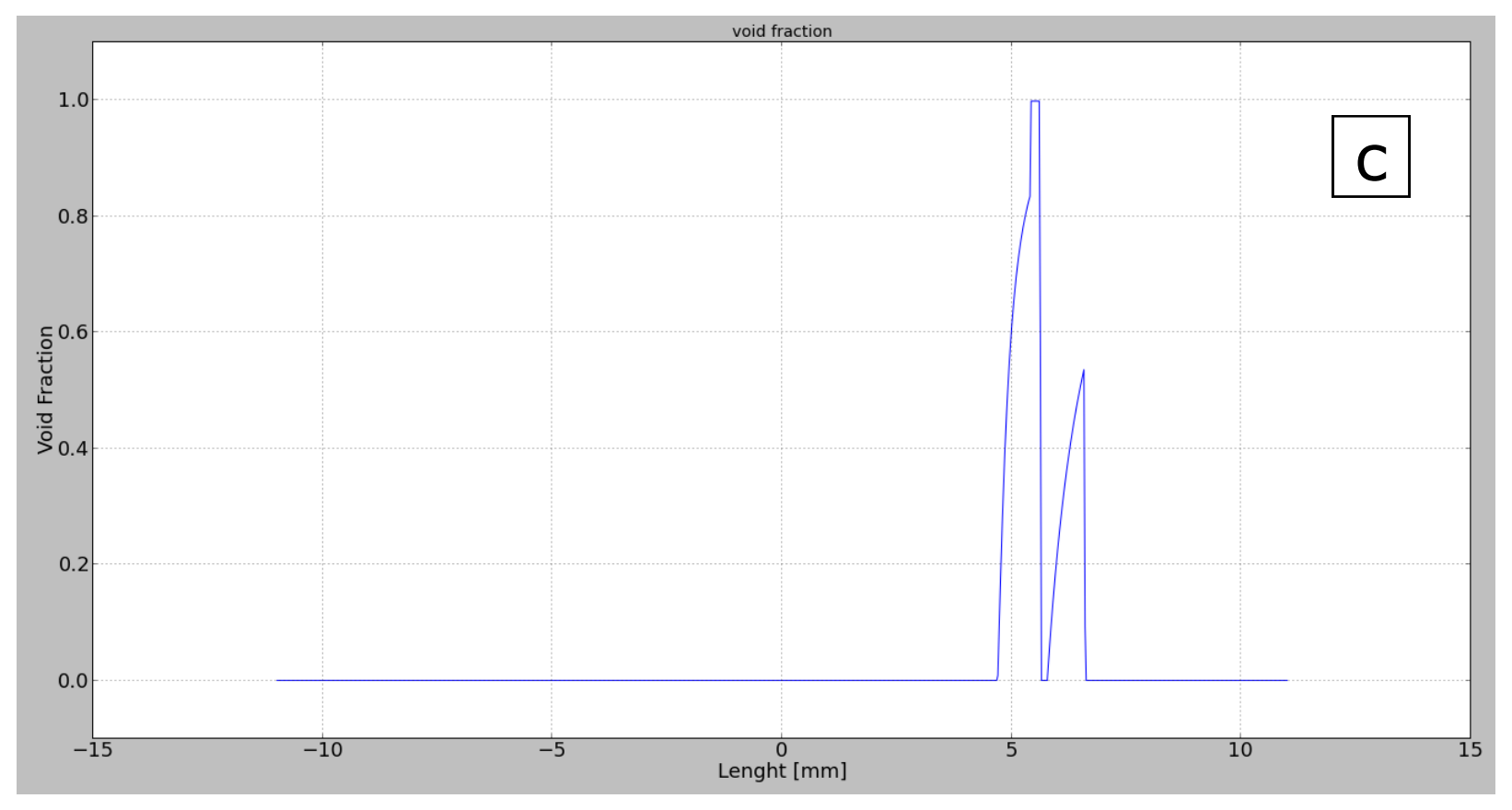
$$\begin{gathered}p_d+[K]h_d=[K]h_g\\ p_d\geq 0\\ h_d\geq 0\\ p_d\cdot h_d=0\end{gathered}\quad\Longrightarrow\quad h=h_g+h_e+h_d$$



The influence of texture height and width is analyzed in terms of friction force in a steady state ball-on-plane configuration (constant force 1500N and constant speed 28mm/s). The results show a trend of the friction force to decrease with the increase of the texture height until a threshold is reached beyond which the height has virtually no effect on the friction force. Moreover, the influence of the texture width on the friction force shows a downward trend of friction force with the increase of the width of textures.







The Figures (a) (b) and (c) show the profile of pressure, film thickness and void fraction, respectively, for the textured surface of height of 0.05mm and of width of 0.3mm. The Figure (a) shows the pressure profile; it is important to underline that into the texture the pressure remains almost constant. The film thickness profile (b) shows a trend similar to the known one in the field of hip EHL. In addition the increase of film over the textures is visible. The Void Ratio profile (c) details the presence of cavitation not only at the end of the lubricated interval, but also in the last texture. This suggests that textured surfaces may be influential on hip lubrication and friction torque reduction. These results are preliminary, but nevertheless they suggest that textured surfaces may have a positive effect on hip lubrication. Moreover the numerical simulation seems to constitute a useful design tool for correctly developing textured surfaces.

M. Giacopini et al "A Mass-Conserving Complementarity Formulation to Study Lubricant Films in the Presence of Cavitation", ASME J. Tribology, 132, 4, 2010, 041702

L. Gao et al "Effect of surface texturing on the elastohydrodynamic lubrication analysis of metal-on-metal hip implants", Tribology International 2010; 43, 1851-1860

L. Bertocchi et al. "Fluid film lubrication in the presence of cavitation: a mass-conserving two-dimensional formulation for compressible, piezoviscous and non-Newtonian fluids", Tribology International 2013; 67, 61-71

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