Analytical-numerical peak contact pressure in a rectangular elastomeric seal with rounded edges achieved by asymptotically matching an available Boussinesq solution with a purposely developed stress intensity factor

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Rectangular seals constitute an alternative design to O-rings. Rectangular seals are employed in demanding applications such as aircraft actuators, e.g. ref. [1]. The seal edges are generally rounded, ref. [2]. As a consequence of the presence of filleted edges, the contact pressure exhibits Hertzian-type local bumps in its lateral zones; it remains almost flat in the central zone of the contact. The lateral bumps and the central flattish zone confer to the contact pressure distribution a camel-backed profile, see ref. [2], and ref. [3] for a similar axisymmetric problem.

It is difficult to derive a rigorous, analytical expression of the contact pressure curve for the title problem. In fact, the analytical solution available for a rectangular punch with rounded edges indenting a half plane, e.g. ref. [4] and related bibliography, is exact only in the situation of a rigid punch indenting a deformable half plane, ref. [5], whereas in the title problem the punch (i.e., the seal) is flexible and the half plane (i.e., the counterface) is rigid.

It has recently been shown in refs [5-7] that the unrealities of the above analytical solution may be corrected by combining the analytical solution with Fracture Mechanics (FM) results dealing with the stress singularities at the tip of a transverse crack in a strip of finite width. In this paper, an extension of formula (20) of ref. [5] is developed, that accounts for the combined effects of a) the presence of a filleted edge, and b) a finite seal width and height.

1. Purposely developed FM solution

The unrealities of the analytical solution of ref. [4] may be corrected by performing an asymptotic matching between two asymptotic solutions, namely a) the analytical pressure profile valid for a frictionless, plane, rounded indenter of semi-infinite width, and b) the corresponding pressure profile describing the sharp-edged equivalent of the problem under scrutiny.

The sharp edged model should account for two main peculiarities of the seal deformation. First, it should account for the effect of the seal material protruding from the contact region. Secondly, it should account for two parts of the rectangular seal border remaining rectilinear under compression, namely a) the flat portion of the sealing profile, and b) the seal opposite side. These two properties may be mimicked by a purposely developed FM solution referring to a cracked strip of finite width, and by imposing that the strip exhibits two transverse symmetry axes, one axis passing through two collinear edge cracks, and a second axis parallel to the previous one and at a distance equal to the seal height. A suitable FM model is therefore an infinitely long strip of finite width under an imposed elongation, exhibiting an infinite array of equispaced, transverse, collinear edge cracks. The above FM problem has not been treated in the pertinent literature, see ref. [8], p.

285 for a similar case. A FM solution has therefore been developed with the aid of FE.

Three singularities are considered, namely a) the presence of lateral collinear cracks, whose length is *a*; b) the presence of a central ligament, which may become vanishingly small, and whose half extent is denoted by *c*; c) the distance *2h* between two parallel, contiguous cracks. The width *w* of the cracked strip is therefore *2(a+c)*.

To derive a formula expressing the classical FM parameter K_I, following e.g. ref. [8], an equivalent length l_{eq} is introduced together with the three auxiliary variables α,β,γ:

$$\frac{1}{l_{eq}}=\frac{1}{a}+\frac{1}{c}+\frac{1}{h};\alpha=\frac{l_{eq}}{a};\beta=\frac{l_{eq}}{c};\gamma=\frac{l_{eq}}{h}\,;\,K_I=\left(\mathrm{C}_1+\mathrm{C}_2\ \alpha+\mathrm{C}_3\ \beta+\mathrm{C}_4\ \gamma\right)\frac{\delta}{h}E'\sqrt{\pi l_{eq}} \tag{1}$$

where δ is the elongation and *E'* is an equivalent elastic modulus, and the numerical coefficients of the bracketed polynomial have been calibrated with a Finite Element (FE) analysis.

2. Asymptotic matching

By performing an asymptotic matching, the following formula for the maximum contact pressure is obtained:

$$p_{\max}\cong C\sqrt[3]{\frac{9\pi K_I^2 E}{6r}} \tag{2}$$

where *r* is the radius of the seal rounded edges, and *C* is a numerical constant. Since it is too difficult to derive analytically the applicability range of the previous formula, an error analysis is under development versus FE forecasts, involving more than one thousand geometries; preliminary, promising results are presented, that indicate a maximum error of about 20 per cent.

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